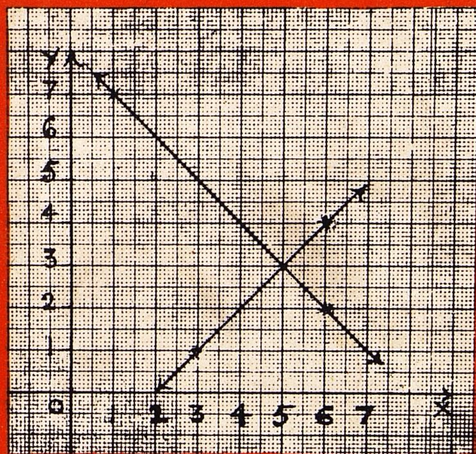
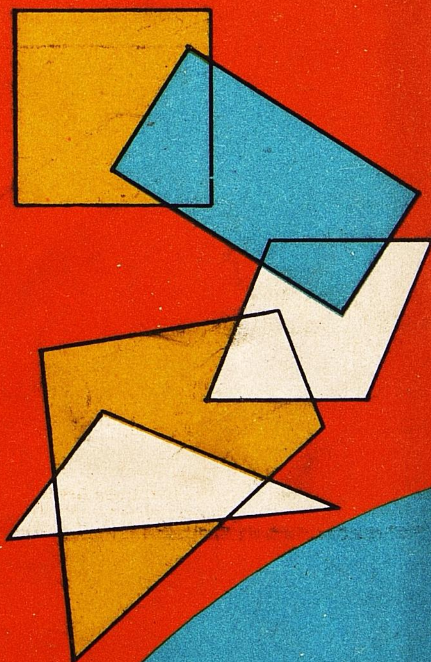
MATHEMATICS 9



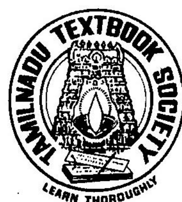




TAMILNADU
TEXTBOOK SOCIETY

MATHEMATICS

Standard IX



TAMILNADU
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MADRAS

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CONTENTS

1. SETS

1. Revision

You would have seen bunches of flowers in gardens. Coconuts will be displayed in heaps in markets. We can see flock of birds in the early mornings. You have a collection of books and note books in your bags. Similar collections of objects are called SETS in the language of Mathematics.

The attendance roll of a class, the foot ball team, a bunch of keys, a collection of postal stamps are some other examples for set.

A part of the attendance register of a class is given below The presence and the absence of a pupil are marked by the symbols × and O respectively.

	M	T	W	T	F
Raman	×	×	×	×	×
Abdul	×	O	×	O	×
Moses	O	×	×	×	×
Kesavan	×	×	O	O	O

Raman, Abdul and Kesavan are the boys who attended the school on Monday. We denote this set of boys by writing their names within a double bracket as shown below.

{Raman, Abdul, Kesavan}

Raman, Abdul and Kesavan are called the members or the elements of this set.

Sets are generally denoted by capital letters.

M = {Raman, Abdul, Kesavan}

Raman is an element of set M. But Moses is not an element of the set M. This can be written as,

Raman $\in$ M; Moses $\notin$ M

'$\in$' means 'belongs to' or 'is a member of'.

'$\notin$' means 'does not belong to' or 'is not a member of'.

If we are able to say definitely an element belongs to a collection or not, then that collection is said to be a well-defined collection.

If we are given a set, then there are only two possibilities. Either an object is an element of the set or not.

If M is a set and 'a' is any object then either $a \in M$ or $a \notin M$.

A SET IS A COLLECTION OF WELL-DEFINED OBJECTS

Notation :

The set E = {0, 2, 4, 6, ...} can be read as E is the set of numbers 0, 2, 4, 6, ... This is a set of even integers. This method of describing a set through listing the members of the set is called the **Roster method**

If the same set is written as

E = {Even integers}

this is known as **descriptive method.**

The third way of describing a set is

E = {n/n is an even integer}.

This is read as E is the set of all n such that n is an even number. This form of writing a set is known as **Set builder form.**

Singleton — Empty set

A = {n/n = 1} is a set which contains only one element. This can also be written as A = {1}.

A set with only one element is called a **singleton.**

Let us consider the set A = {n/n, an odd number and further an even number}

We know that a number can either be an odd number or an even number, but cannot be both at a time. Therefore there will be no element in the set A.

This is an empty set. This can also be written as

$A = \{\ \}$ or $A = \phi$. ϕ is read as 'phi'

Exercise 1

1. Write down the elements of the following sets:
   (a) The set of the letters in the word 'Father'.
   (b) The set of odd whole numbers less than 10.
   (c) The set of prime numbers less than 20.
2. Find out the set of whole numbers divisible by 9 and less than 99.
3. Write down the set of whole numbers whose sum is from the set of whole numbers less than 99.
4. Rewrite the following sets in set builder form.
   (a) $K = \{5, 10, 15, 20, 25\}$
   (b) $M = \{3, 6, 9, 12, 15, ...\}$
   (c) $R = \{a, e, i, o, u\}$
   (d) S = {Raman, Ragavan, Radhakrishnan}

2—1. Subset

We can write the set of boys in a bench as $A = \{a, b, c, d, e\}$. Among them let the set of boys who attended the school on a particular day be

$$P = \{a, b, c\}$$

Each and every element of P is an element of A. P is called a subset of A. This is symbolically written as $P \subset A$.

If $P \subset A$, then $P \neq A$. Therefore P is a proper subset of A.

All the elements of the set $\{a, b, c, d, e\}$ are also the elements of the set $A = \{a, b, c, d, e\}$.

Therefore the set itself is a subset of the set. The empty set is considered a subset of all sets. The set itself and the empty set are called improper subsets.

If S is the set of all boys of the class then,
$P \subset A \subset S$

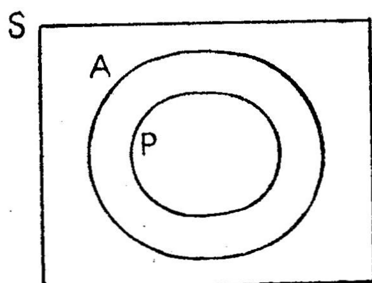


Fig. 1-1

Consider the set A = {a, b}

Write down the subsets of A.

{a, b}, {a}, {b}, { } are the subsets of A.

In these subsets {a, b} is the improper subset, and { } is the empty set.

The set itself and the empty set are subsets of each and every set.

Complete the following table :

Set	No. of elements	Subsets	No. of subsets
{a}	1	{a} , { }	2
{a, b}			
{a, b, c}			

If the set contains only one element, then the number of subsets will be 2.

If the set contains 2 elements, then the number of subsets will be 4.

How many subsets are there in a set if it contains 3 elements ?

We can conclude from the table that the set of 3 elements has 8 subsets.

Thus we can infer that the number of subsets will be 2, 4, 8 or 2, 2^2, 2^3 for the sets containing 1, 2, 3 elements respectively.

Verify: The set of 4 elements has 2^4 (= 16) subsets. Therefore we can conclude that

> If 'n' is the number of elements in a set, then the total number of subsets will be 2^n.

Power set

A = {a, b}

The subsets of A are {a, b}, {a}, {b}, { }.

If we form a set of these subsets, then it can be written as

P = { {a, b}, {a}, {b}, { } }

This is called the **power set** of A.

P (A) = { {a, b}, {a}, {b}, { } }

If A is a set,

P (A) = $\{x/x \subset A\}$.

> Thus the set of all the subsets of a set is the power set of that set.

Point to be noted: The elements of a power set are also sets.

Exercise 2—1

1. Write down the subset of Prime numbers of the set A = {1, 3, 5, 7, 9}.
2. How many subsets are there in the above question?
3. Insert $\subset$ or $\not\subset$, as the case may be.

   (a) {1, 2, 3} — {3, 1, 2}

(b) $\{2, 4, 6, 8, 10\}$ — $\{6, 3, 8, 12, 10, 4\}$

(c) $\{a, e, i, o, u\}$ — {x/x is an English Alphabet}

(d) {x/x is a multiple of 10} — {x/x is a multiple of 5}

(e) {x/x is a prime number lying between 10 and 100} —{x/x is an even number lying between 10 and 100}

Answers

1. $\{1, 3, 5, 7\}$ 2. 32 3. (a) $\subset$ (b) $\not\subset$ (c) $\subset$ (d) $\subset$ (e) $\not\subset$

2—2. Intersection of sets

$M = \{a, b, c\}$; $T = \{a, c, d\}$

where M is the set of Volley ball players who played on Monday and T is the set of players who played on Tuesday.

If S is the set of players who played on both days, then $S = \{a, c\}$

That is, S is the set whose elements are common to both the sets M and T. The set S is called the **intersection** of the sets M and T.

This is symbollically written as

$$S = M \cap T$$

Find $M \cap T$ and $T \cap M$.

What can we presume from this ?

We can see that $M \cap T = T \cap M$.

Verify this result for various sets.

If A and B are any two sets, then $A \cap B$ is the set of elements common to both A and B.

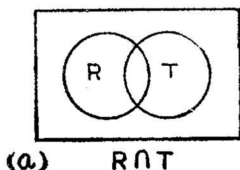
$A \cap B = \{x/x \in A \text{ and } x \in B\}$.

Exercise 2—2

1. Write down the intersection of the sets $\{a, b, c, d, e\}$ and $\{a, e, i, o, u\}$.

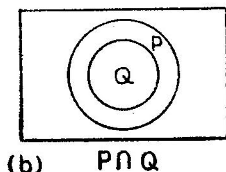
2. Write down the intersection of the set of Prime numbers less than 10 and the set of odd numbers less than 10.

3. Shade the intersection of the sets.



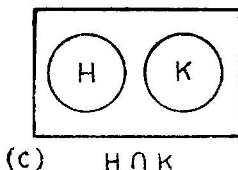
(a) $R \cap T$

Fig. 1-2



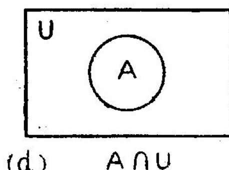
(b) $P \cap Q$

Fig. 1-3



(c) $H \cap K$

Fig. 1-4



(d) $A \cap U$

Fig. 1-5

4. Find the intersection of the following sets.

(a) $A \cap A$ (b) $A \cap \phi$ (c) $A \cap U$.

Answers

(1) {a, e} (2) {1, 3, 5, 7}

(4) (a) A (b) ϕ (c) A.

2—3. Union of two sets

T = {b, c, e} ; F = {a, b, c}

where T is the set of students who attended the school on Thursday and F is the set of students who attended the school on Friday.

If K is the set of students who came to school on any one of the days, then the elements of K will be all the elements of T and F.

$a \in K$; $b \in K$; $c \in k$; $e \in K$.

It can be seen that d is not an element of K. Note $d \notin T$, $d \notin F$.

$K = \{a, b, c, e\}$

The set K is the **union** of the sets T and F and is denoted as $K = T \cup F$

If $M = \{c, e, f\}$; $N = \{c, r, l\}$ then

$$M \cup N = \{c, e, f, r, l\}$$

If A and B are any two sets, then $A \cup B$ is the set of elements contained either in A or in B.

$A \cup B = \{x / x \in A \text{ or } x \in B\}$.

This is illustrated in the adjoining **Venn diagram.**

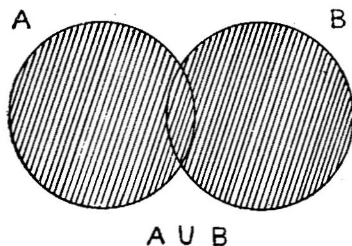


Fig. 1—6

Exercise 2—3

1. (a) $A = \{a, b, c, d\}$; $B = \{b, d, f, h\}$

   $A \cup B = ?$

   (b) $x = \{2, 4, 6, 8, 10\}$; $y = \{3, 6, 9, 12\}$

   $x \cup y = ?$

   (c) If $A \subset B$, then find $A \cup B$.

2. Shade in the Venn diagram

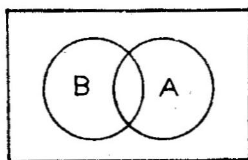


Fig. 1—7

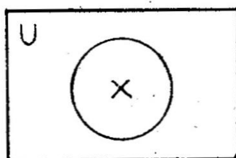


Fig. 1—8

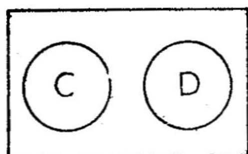


Fig. 1—9

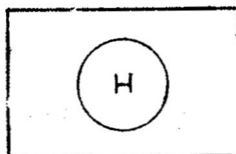


Fig. 1—10

Answers

1. (a) { a, b, c, d, f, h } (b) { 2, 3, 4, 6, 8, 9, 10, 12} (c) B

2—4. Properties of Sets

Commutative Property

If $A = \{1, 2, 3\}$ and $B = \{3, 4, 6\}$, find $A \cup B$, $B \cup A$, $A \cap B$, $B \cap A$.

Find out the nature of $A \cup B$ and $B \cup A$.

Find out the nature of $A \cap B$ and $B \cap A$.

Take some more sets and continue the experiment.

What can be inferred from these experiments?

The union and the intersection of two sets obey the Commutative property, that is,

$$A \cup B = B \cup A; \quad A \cap B = B \cap A$$

If $A = \{a, b, c, d\}$; $B = \{c, d, e, f\}$; $C = \{a, e, g, h\}$ find $B \cup C$; $A \cap (B \cup C)$.

Also find $A \cap B$, $A \cap C$.

Compare the results $A \cap (B \cup C)$; $(A \cap B) \cup (A \cap C)$

From this we can infer that

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

This property is illustrated in Fig. 1—12 with the help of Venn diagrams.

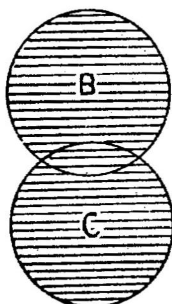


Fig. 1—11

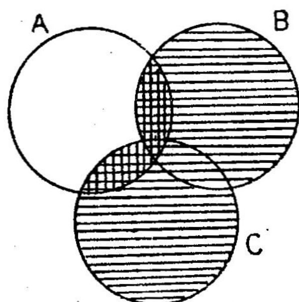


Fig. 1—12

The portion shaded with crossed lines represents

$A \cap (B \cup C)$

If $A = \{1, 2, 3\}$; $B = \{3, 4, 6\}$ and $C = \{2, 5, 6, 7\}$

then find $A \cap B$ and $A \cap C$.

Also find $(A \cap B) \cap (A \cap C)$

Also find $(A \cap B \cap C)$

What can be inferred from these results ?

$$(A \cap B \cap C) = (A \cap B) \cap (A \cap C)$$

Find the following:

(a) $B \cap C$ (b) $A \cup (B \cap C)$ (c) $A \cup B$; $A \cup C$

(d) $(A \cup B) \cap (A \cup C)$

Find out the relation between $A \cup (B \cap C)$ and $(A \cup B) \cap (A \cup C)$

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

Exercise 2—4

1. Find the following:

   (a) {x/ x is an even number} $\cup$ {x/ x is an odd number}

   (b) {x/ x is a mu tiple of 2 } $\cup$ {x/ x is a multiple of 3}

   (c) {x/ x is a negative integer} $\cup$ {x/ x is a positive integer}

2. It $A \subset B$, prove that $B \cup A = B$.

3. If $A \subset B$, prove that $B \cap A = A$.

4. If N is the set of all natural numbers, A is the set of all odd natural numbers, B is the set of all even natural numbers and C is the set of all multiples of 3, then find the following:

   (a) $A \cap B$ (b) $A \cap C$ (c) $B \cap C$ (d) $N \cup A$ (e) $N \cap B$ (f) $A \cap B \cap \phi$

Answers

1. (a) The set of whole numbers

   (b) {2, 3, 4, 6, 8, 9, 10, ...}

   (c) {..., —3, —2,—1, 1, 2, 3, .. }

4. (a) ϕ (b) { 3, 9, 15, 21, ...} (c) {6, 12, 18, ...}

   (d) A (e) B (f) ϕ

2—5. Complement of a set

Let E be the Universal set and A be a subset of it. The set of elements not in A but in E is called **complement of set A or complementary set of A** and is denoted by A′.

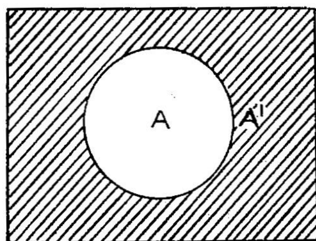


Fig. 1-13

Example 1 :

If E = {1, 2, 3, 4, 5, 6, 7, 8} and

A = {1, 3, 5, 7, 8} then

A′ = {2, 4, 6}

Example 2 :

If E = {x/x is an English Alphabet} and

A = {a, e, i, o, u} then

A′ = {x/x is a consonant}

If E is the given Universal set, then

A′ = {x/x $\in$ E and x $\notin$ A}

If E = A, then A′ = E′ = ϕ

Exercise 2—5

1. If N is the set of natural numbers

A is the set of odd natural numbers

B is the set of even natural numbers and

C is the multiples of 3 find

(a) C′ (b) A′ (c) B′

2. Draw Venn diagram for the following:

(a) $(A \cup B)'$ (b) $A' \cap B'$ (c) $(A \cap B)'$ (d) $A' \cup B'$

Answers

1. (a) $C' = \{x/x \in N;\ x \notin C\}$

(b) $A' = \{x/x \in N;\ x \notin A\}$

(c) $B' = \{x/x \in N;\ x \notin B\}$

2—6. Cardinal number of a set

If $A = \{1, 2, 3, 4\}$, the number of elements of this set is 4. That is the Cardinal number of the set A is 4.

It is denoted as $n(A) = 4$

$A = \{x/x$ is an even number $< 9;\ x \in N\}$

$A = \{2, 4, 6, 8\}$

$n(A) = 4$

If $A = \{1, 3, 4, 5\}$ and $B = \{2, 6, 8\}$ find $n(A \cup B)$.

Find the relation between $n(A)$, $n(B)$, $n(A \cap B)$ and $n(A \cup B)$

If $A = \{1, 4, 5, 6\}$ and $B = \{2, 4, 8\}$ find $n(A \cap B)$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Solution through Venn diagram

Example : There are 24 teachers teaching Mathematics and Science in a school. 14 teachers teach Mathematics and 5 teachers teach both Mathematics and Science. Find the number of teachers who teach Science.

M — the set of teachers who teach Mathematics.

S — the set of teachers who teach Science.

$M \cap S$ — the set of teachers who teach both mathematics and Science.

Represent these in Venn diagram

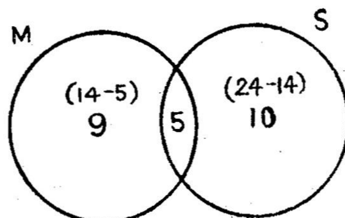


Fig. 1-14

$n(M) = 14; n(M \cap S) = 5; n(M \cup S) = 24$

$n(S) = 10 + 5 = 15$

Exercise 2—6

1. In a school 21 students are learning carpentry and 17 students are learning weaving. Out of these students 12 are learning both weaving and carpentry. Find the total number of students who learn either carpentry or weaving or both.

2. In a school 25 students can play foot ball, 15 can play Hockey and 7 can play both. Find the number of students

   (i) who can play only foot ball

   (ii) who can play only hockey

   (iii) total number of students who can play either one of the above games.

3. There are 40 houses in a street. A newspaper agent sells 27 Hindu papers and 15 Indian Express papers there. If none buys more than two dailies find the least and highest number of houses who buy both the dailies.

Answers

1. 26 2. (i) 18; (ii) 8; (iii) 33 3. 2; 15

3—1. Probability (1)

If a fair coin is tossed we will get either a head or a tail.

A die is marked with numbers 1, 2, 3, 4, 5 and 6 on its faces. If it is rolled, we will get either one of 1, 2, 3, 4, 5 and 6.

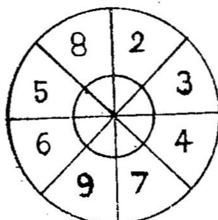


Fig. 1-15

A dart board is shown in Fig. 1-15.

If we aim a pin at the board, where will it hit ?

Tossing a coin, rolling a die, throwing an arrow, taking any one by lots and similar examples are called **Experiments.**

Getting a tail, getting 5 in a die, hitting 8 by throwing an arrow are called **Out comes or Events.**

The set of all possible outcomes of an experiment is called a **Sample Space.**

Example 1:

The sample space of tossing a coin = { H, T}

The sample space of rolling a die = { 1, 2, 3, 4, 5, 6 }

The sample space of throwing an arrow

= {2, 3, 4, 5. 6, 7, 8, 9}

Example 2 :

The sample space of choosing a ball from a bag of 3 yellow and 2 green balls = $\{y_1, y_2, y_3, G_1, G_2\}$

Example 3 :

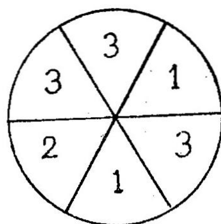


Fig. 1-16

The sample space of rotating the adjoining card

= $\{1_1, 1_2, 2, 3_1, 3_2, 3_3\}$

Exercise 3-1

Give the sample space for the following experiments:

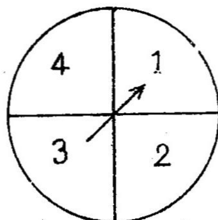


Fig. 1-17

1. Rotating the pin in the adjoining card.

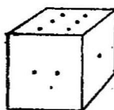


Fig. 1-18

2. Rolling a die as shown in the adjoining figure.

Answers

1. S = {1, 2, 3, 4}

2. S = {1, 2, 3, 4, 5, 6}

3—2. Probability (2)

We saw that {H, T} is the sample space in tossing a coin. The two possibilities of outcomes are {H} and {T}.

If the coin is a fair one these possibilities are having equal chances. These chances are known as the probability of an event in an experiment. The probability of getting a head is $\frac{1}{2}$; The probability of getting a tail is $\frac{1}{2}$.

Similarly the numbers ranging from 1 to 6 are having equal chances of coming up in rolling a die.

The probability of getting 3 in rolling a die is $\frac{1}{6}$.

If S is the sample space and E is an event, then the probability of an event E is defined by

$$P(E) = \frac{n(E)}{n(S)}$$

Example: There are 6 red marbles and 4 blue marbles in a bag. If a marble is chosen at random from that bag, determine the probability that it is (a) a red marble (b) a blue marble.

$S = \{R_1, R_1, R_3, R_4, R_5, R_6, B_1, B_1, B_3, B_4\}$.

$(R_1, R_1, \ldots R_6)$ represent the red marbles, and

$(B_1, B_1, \ldots B_4)$ represent the blue marbles.

$$n(S) = 10.$$

Event 1 is choosing a red marble.

$E_1 = \{R_1, R_2, R_3, R_4, R_5, R_6\}$

$n(E_1) = 6$

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{6}{10} = \cdot 6$$

Event 2 is choosing a blue marble.

$E_2 = \{B_1, B_2, B_3, B_4\}$

$n(E_2) = 4$

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{4}{10} = \cdot 4$$

Exercise 3–2

1. A cube is marked with A, B, C, D, E, F on its faces. Find the sample space when it is rolled. Find the probability of getting each of the letters.

2. Give the sample space when the needle in the disc is rotated.

(a) Find the probability of getting 1.

(b) Find the probability of getting 3.

(c) Find the probability of getting 2.

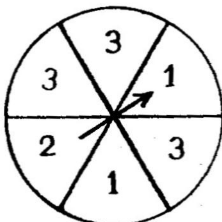


Fig. 1–19

3. There are 4 red baloons and 5 black baloons in a bag. If a baloon is chosen at random, find the probability of

(a) getting a red baloon.

(b) getting a black baloon.

4. In sum 3 above if two baloons are taken at random at the same time determine the probability of the following events:

(a) both are red in colour.

(b) both are black in colour.

(c) one red and one black.

Answers

1. $\frac{1}{6}$ 2. (a) $\frac{1}{3}$, (b) $\frac{1}{2}$, (c) $\frac{1}{6}$

3. (a) $\frac{4}{9}$ (b) $\frac{5}{9}$

4. (a) $\frac{1}{6}$ (b) $\frac{5}{18}$ (c) $\frac{5}{9}$

3–3. Probability—Experimental Probability

So far we have studied the Mathematical Probability. We know that getting a head and getting a tail are two events having equal chances in tossing a coin and the probability of each outcome is $\frac{1}{2}$.

We cannot foretell whether a tail follows a head or not. It may again be a head.

But if the experiment is repeated a number of times, the two events, that is, getting a head and getting a tail may almost have equal chances.

If we get the head 484 times and the tail 516 times in tossing a coin 1000 times then,

the experimental probability of getting a head $= \frac{484}{1000}$

the experimental probability of getting a tail $= \frac{516}{1000}$

Calculate the number of events when each boy tosses a coin 10 times. Find the experimental probability considering all the experiments as one single experiment.

Then take a bag with 5 yellow, 3 red and 2 white marbles. Choose one marble and note down its colour. Let the students continue the experiment one after another.

Repeat the experiment 1 time, 2 times, ... 10 times. Find the experimental probability of these experiments.

Experimental probability will approach theoretical or mathematical probability as the experiment is repeated a large number of times.

4. Pair — Ordered pair

Two friends went to a hotel. They wanted to take one sweet and one savoury from the set of Laddu, Jangiri, Vadai and Mixture in the order of a sweet followed by a savoury.

Write down all the possibilities. Each possibility is an ordered pair. The first element of the pair is a sweet and the second element is a savoury.

To identify the place of a boy in a class, we can say that he is the third boy from the right in the second row. If we represent the row and the place in a pair we can write the position of Raman as (2, 3). In this pair the first element represents the number of rows and the second element represents his position from the right. This is an ordered pair.

(3, 2) represents the boy who is the second from the right in the third row. This pair will not refer to Raman. It shows the place of some other boy.

Fraction — an ordered pair:

$\frac{2}{3}$ is a fraction. In this fraction 2 is the numerator and 3 is the denominator. This can be written as (2, 3). Does (3, 2) represent the same fraction or a different fraction ? Why?

In this manner we can represent all fractions. Find the addition and multiplication property of this ordered pair.

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

This can be written as (ad + bc, bd)

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$$

$\therefore$ (a, b) × (c, d) = (ac, bd)

Note : The notation for an ordered pair is (x, y)

Integers as ordered pairs :

An integer can be written as a difference of two whole numbers.

$$4 \equiv 6-2$$

$$-4 \equiv 2-6$$

Hence the ordered pair (6, 2) represents the integer 4 and the ordered pair (2, 6) represents the integer —4. We are subtracting the second number from the first number.

Since 4 = 6—2 ≡ 8—4 ≡ 10—6 ≡ 12—8 ≡ 5 — 1 the ordered pairs (6, 2), (8, 4), (10, 6), (12, 8) and (5, 1) all represent the same integer 4.

In the ordered pair of integers,

Addition rule:

$$(a, b) + (c, d) = (a + c, b + d)$$

Multiplication law :

$$(a, b)\,(c, d) = (ac + bd, ad + bc)$$

Verification of these results.

Example : —4 ≡ (2 , 6)

—3 ≡ (1 , 4)

$(-4) + (-3) \equiv (2 + 1, 6 + 4) = (3, 10) = -7$

$(-4) \times (-3) = (2 + 24, 8 + 6) = (26, 14) = 12$

Verify the above laws with some other sets of integers.

Cartesian product :

$$A = \{1, 2\}$$

$$B = \{3, 4, 5\}$$

Now we can form the ordered pairs with the elements of these two sets. If we associate each element of A with each element of B, we get (1, 3), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5). The set of these ordered pairs is called the cartesian product of the sets A and B.

The first element of these ordered pairs is the element of set A and the second one is the element of set B. This is written as A × B and read as A cross B.

A × B is also called the product set.

$A \times B = \{(1, 3), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5)\}$.

Note that A × B is a set and its elements are ordered pairs.

The set builder notation of this set is

$$A \times B = \{ (x, y) \,/\, x, \in A;\, y \in B\}.$$

This can be shown using a tree diagram or in a graph.

Tree diagram :

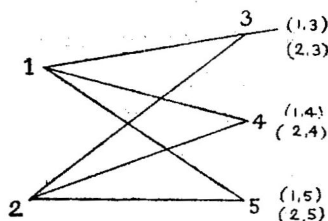


Fig. 1-20

Graph :

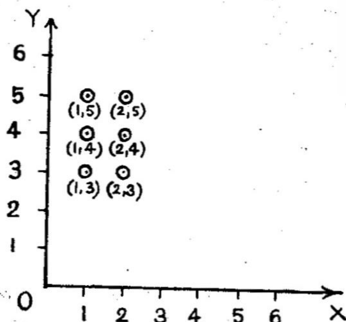


Fig. 1-21

Exercise 4

1. If $A = \{m, n\}$; $B = \{x, y\}$ and $C = \{r, s, t\}$ then find the following:

   (a) $A \times B$ (b) $A \times C$ (c) $C \times B$
   (d) $n(A \times B)$ (e) $n(B \times C)$

2. Is $A \times C = C \times A$ a true statement ?

3. $A = \{1, 2, 3\}$ Find $A \times A$ and represent it in a graph.

Answers

1. (a) $\{(m, x), (m, y), (n, x), (n, y)\}$

   (b) $\{(m, r), (m, s), (m, t), (n, r), (n, s,), (n, t)\}$

   (c) $\{(r, x), (r, y), (s, x), (s, y), (t, x), (t, y)\}$

   (d) 4 (e) 6 2. No.

3. $\{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$

5—1. Domain — Co-domain—Range :

The daily attendance of some boys are given below:

Name	M	T	W	TH	F
a	×	O	×	×	×
b	O	O	×	×	×
c	×	×	×	O	×
d	×	×	O	×	×

The set of boys = { a, b, c. d}

The set of notations = { ×, O }

We can represent the above sets in diagrams.

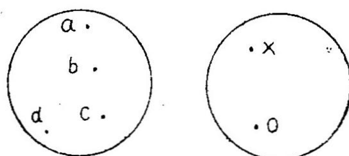


Fig. 1-22

The symbol pertaining to the boys a, c, d on Monday is X and that of b on Monday is O.

How can this be represented in the diagram ?

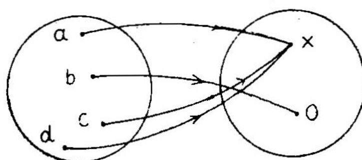


Fig. 1-23

We can draw diagram for other days also.

The data represented in the diagram can be represented by ordered pairs. Each boy should be associated with the respective symbol.

(a, ×), (b, o), (c, ×), (d, ×)

The first element of each pair represents the boy and the second element represents the symbol, denoting his presence or absence.

We can see a definite relation between the first and the second element in the above pairs.

The set of ordered pairs is { (a, ×), (b, o), (c, ×), (d, ×)}

Note down the set of drinks you and your friend have taken on a particular day.

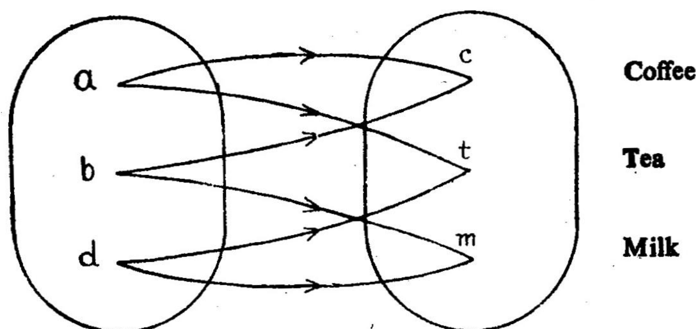


Fig. 1-24

This can be written in ordered pairs as

{ (a, c), (a, t), (b, c), (b, m), (d, t), (d, m)}

The first elements of the ordered pairs represent the persons who have taken drinks and the second elements represent the drinks they have taken.

The set of the first elements of these ordered pairs is called the **Domain**, that is, { a, b, c} is the Domain.

The set of the second elements of the ordered pairs is called its **Range**, that is, {c, t, m} is the Range.

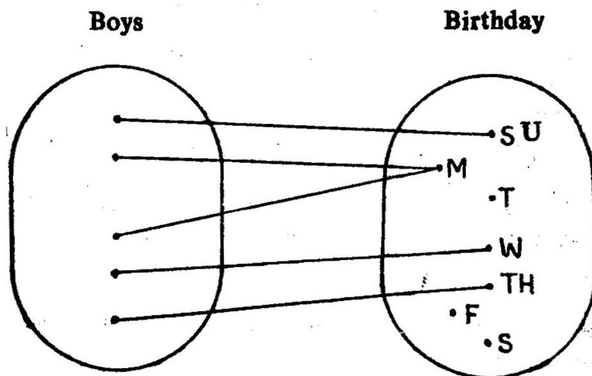


Fig 1-25

What does this figure represent?

More than one boy may have the same day as birthday. It is also possible that no one has a particular day as birthday. In this, the set of boys is the domain, and the Range is = { Su, M, W, Th}

Co-domain = { Su, M, T, W, Th, F, S},

Range is a subset of the co-domain.

Domain → the set of the first elements of the ordered pairs.

Range→ the set of the second elements of the ordered pairs.

We can say the set which has Range as a subset is the co-domain.

Exercise 5—1

1. Write down the Domain and the Range.

(a) $\{(-3, 1), (-1, 1), (1, 0), (3, 0)\}$

(b) $\{(1, 1), (-1, 0), (0, 0), (1, 0), (2, 0)\}$

2. Write down the ordered pairs, the domain and the range in the following problems.

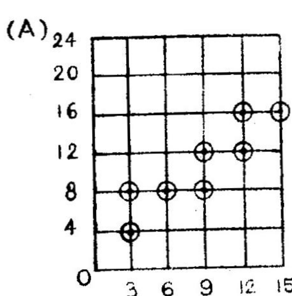


Fig. 1-26

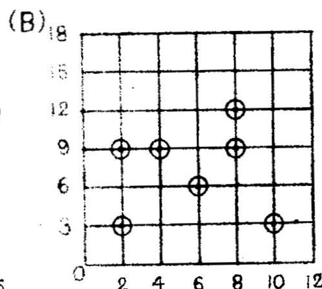


Fig. 1-27

3. Write down the Domains and Ranges in the following :

(a) $\left\{\left(x, \frac{1}{x}\right) / \ 0 < x < 6;\ x \in Z\right\}$

(b) $\{(1, 1), (2, 4), (\sqrt{3}, 3), (3, 9)\}$

Answers

1. (a) $\{-3, -1. 1, 3\}$, $\{1, 0\}$

   (b) $\{1, -1, 0, 2\}$, $\{1, 0\}$

2. (a) $\{(3, 4), (3, 8), (6, 8), (9, 8), (9, 12), (12, 12), (12, 16), (15, 16)\}$

   Domain $\{3, 6, 9, 12, 15\}$; Range $= \{4. 8, 12, 16\}$

   (b) $\{(2, 3), (2, 9), (4, 9), (6, 6), (8, 9), (8, 12), (10, 3)\}$

   Domain $= \{2, 4, 6, 8, 10\}$; Range $= \{3, 6, 9, 12\}$

3. (a) $\{1, 2, 3, 4, 5\}$, $\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}\}$

   (b) $\{1, 2, \sqrt{3}, 3\}$, $\{1, 4, 3, 9\}$

5—2. Functions

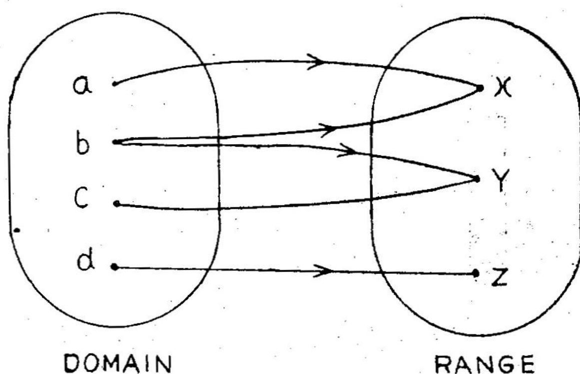


Fig. 1-28

Note that for some elements in the Range, there are two pre-images.

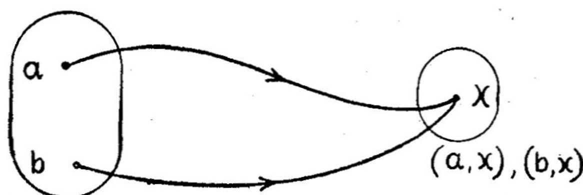


Fig. 1–29

Note that the element of the domain has two images in Fig. 1-30.

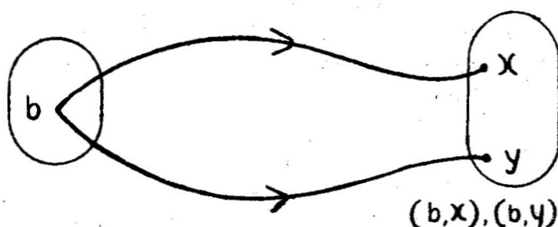


Fig. 1-30

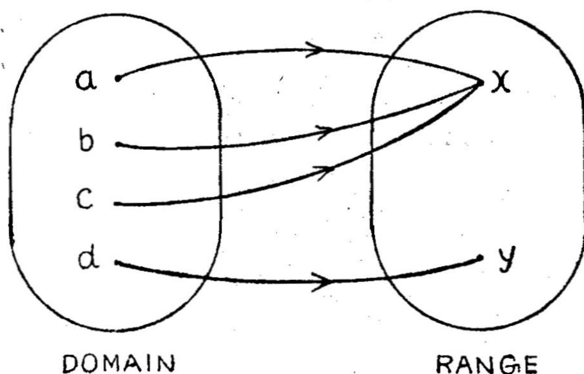


Fig. 1-31

Look at this figure. We can see that one arrow starts from every element of the Domain. Such types of relations are called functions.

$\{ (1, 6), (3, 8), (9, 14) \}$

If we add 5 to the first element of this ordered pair we get the second element.

In the set $\{ (2, 10), (3, 15), (4, 20), (0, 0) \}$

to get the second element, we multiply the first element by 5.

In these two examples, the first elements of the ordered pairs are distinct. If it is so then that relation is called a function.

In a set of ordered pairs, if the first elements are distinct, then it is called a **function**. Note that each and every element of the domain is associated with only one element of the Range.

Let x and y be the first and the second elements in the set of ordered pairs. If for every x, there exists only one y, then that set is called a function. In the diagram only one arrow should start from a point of the domain set. More than one arrow may reach a point in the Range.

Exercise 5—2

1. Which of the following are functions ? Why ?

(a) $\{ (9, 4), (7, 2), (21, 16) \}$

(b) $\{ (9, 3), (6, 2), (2, \frac{2}{3}) \}$

(c) $\{ (0, 0), (3, 3), (7, 7) \}$

(d) $\{ (5, 6), (9, 10), (9, 14) \}$

(e) $\{ (1, 3), (2, 5), (1, -1), (2, -4) \}$

Answers

1. (a), (b), (c) are functions.

(d), (e) are not functions. The first elements are not distinct. The same first element is in more than one ordered pair.

5—3. Graph of a function

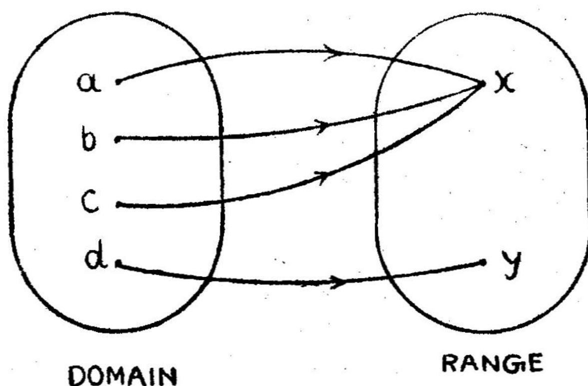


Fig. 1-32

Note that only one arrow starts from each and every element of the Domain, that is, for every element in the domain, there exists only one value in the range.

In the above figure the image of a is x

This can be written as f (a) = x

This can also be written as $a \xrightarrow{f} x$

x = f (b); x = f (c); y = f (d)

Example :

A = { 1, 2, 3, 4 }

Then f is a function of x.

$$f(x) = 3x^2 + 1$$

or

$$x \xrightarrow{f} 3x^2 + 1$$

If $x \in A$, then $f(1) = 3 \times 1^2 + 1 = 4$

$f(2) = 3 \times 2^2 + 1 = 13$

$$f(3) = 3 \times 3^2 + 1 = 28$$
$$f(4) = 3 \times 4^2 + 1 = 49$$

The domain of this function is { 1, 2, 3, 4}

Range is {4, 13, 28, 49}

Exercise 5—3 (a)

1. **If B = {0, 1, 2, 3, 4}; f(x) = 5y + 6; y ∈ B find the Range of this function.**

2. **Find f(0); f(2); f(4); f(6) if C = {x/ x is an odd number} and f(x) = 2x + 1 where x ∈ C.**

3. **If A = {—3, —2, —1, 0, 1} and f(x) = x+1 find its range.**

Answers

1. { 6, 11, 16, 21, 26} 2. 1, 5, 9, 13
3. {—2, —1, 0, 1, 2}

The graph of C :

C={ (0, 0), (1, 1), (2, 2), (3, 3) }

Is C a function ? Why?

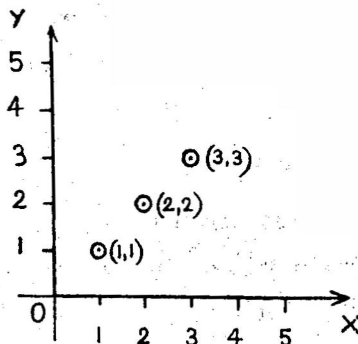


Fig. 1-33

Note that there is not more than one point in any vertical line. In this set there is not more than one ordered pair having the same first element.

Exercise 5—3 (b)

1. State the reasons why the following diagrams do not represent functions.

(a)

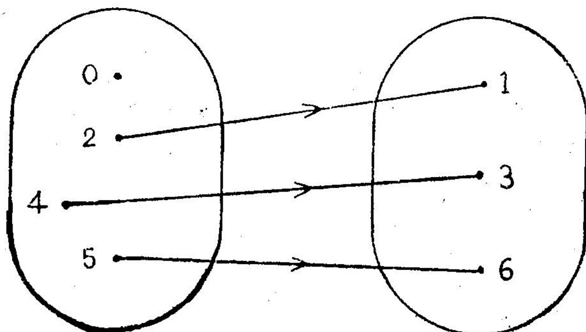


Fig. 1-34

(b)

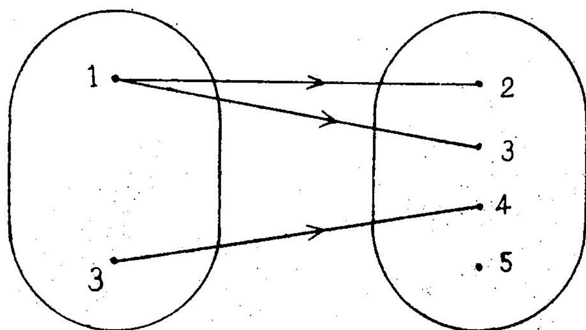


Fig. 1-35.

Answers

1. The element 0 of the Domain is not associated with any element of the Range.

(b) The element 1 of the Domain is associated with more than one element of the Range. (1, 2), (1, 3).

5—4. Constant functions

Find the Domain and the Range in the function.

G = { (1, 2), (2, 2), (3, 2), (4, 2), (5, 2) }

Domain D = { 1, 2, 3, 4, 5 }

Range R = {2}

How will you represent this in an arrow diagram ?

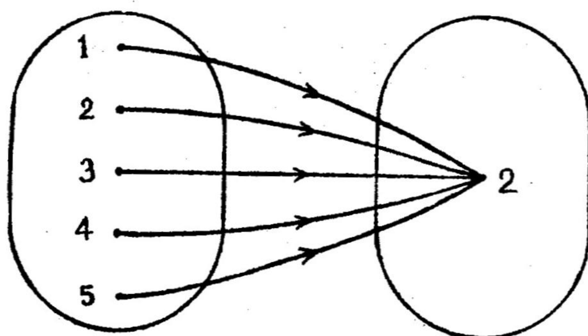


Fig. 1-36

Note that there is only one element in the Range.
It's graph is

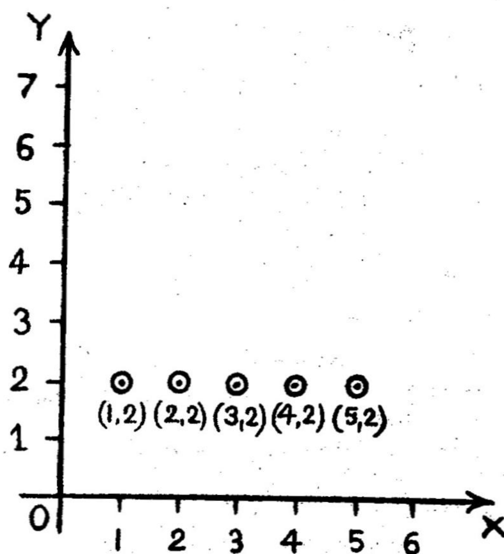


Fig. 1-37

All the elements of the domain are associated with only one element of the range. Such functions are called **constant functions.**

In symbols it is written as $f(x) = a$.

$x \in$ the domain. a is a constant.

5 –5. Direct variation function

Revision :

The ratio between x and y is $x : y$ or $\frac{x}{y}$

The ratio between 6 and 3 is $6 : 3$ or $\frac{6}{3}$

The ratio between 4 and 12 is $4 : 12$ or $\frac{4}{12}$

y	8	12	16	32	64	128	256
x	2	3	4	8	16	32	64

Find the ratio between x and y in the above table.

$x : y = 1 : 4$

i.e. $\frac{y}{x} = 4$ or $y = 4x$

Note that the ratio between x and y is not changing.

Note : $y = f(k)$

If the ratio between two variables is not changing then that function is called a **Direct variate function.**

$$\frac{y}{x} = k \text{ or } y = kx$$

$$\frac{f(x)}{x} = k;\ f(x) = kx$$ where k is a constant.

Example :

Is y = 2x a direct variate function ?

$$y = 2x$$

$$\frac{y}{x} = 2 \quad (x \neq 0)$$

$$\therefore \frac{y}{x} = 2 \text{ is true}$$

$\therefore$ y = 2x is a direct variate function.

The cost of a pen is Rs. 3. The cost of 2 pens is Rs. 6. Tabulate these facts.

Consider the number of pens as x and the cost of pens as y.

y	3	6	9	12	15	18	21	24
x	1	2	3	4	5	6	7	8

$\frac{y}{x} = 3$; y = 3x. This is a direct variable function. If it is represented in a diagram, then we can see that these points will form a straight line (x, y $\in$ **R**)

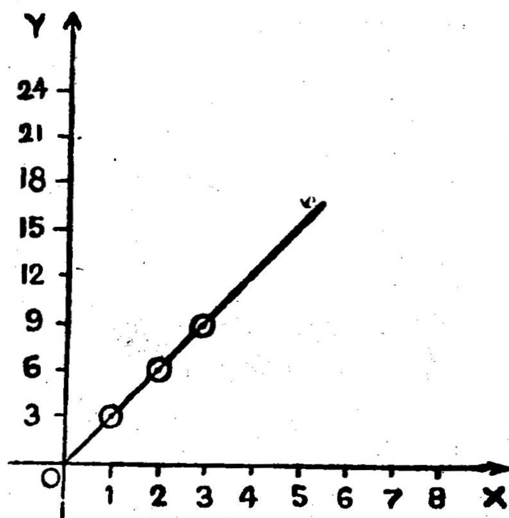


Fig. 1—38.

Exercise 5—5

1. Which of the following are direct variation functions?

(1) $y = 5x$ (2) $y = 3x + 7$ (3) $m = \frac{1}{4} n^2$ (4) $C = 6a$

(5) $A = 2r + 9$ (6) $\frac{C}{d^2} = 7$

2. Which of the following tables represent direct variation functions?

(a)

y	x
1	—2
2	—4
—1	2
—2	4

(b)

c	d
5	2
7	3
11	5
15	7

(c)

A	r
15	3
10	2
20	4
25	5

3. If $x = -4$, $y = 16$, find y when $x = 8$.
4. If $x = 27$, $y = 27$, find y when $x = -6$.
5. A boat travels 120 km in 12 hours. Find the distance travelled by the boat in 18 hours.

Answers

1. (1), (4) are direct variation functions.
2. (a), (c) are direct variation functions.
3. —32 4. —6 5. 180 km.

5–6. Inverse variation function

Look at the table :

x	—16	—2	$\frac{1}{4}$	$\frac{1}{2}$	2	4	8	32
y	—4	—32	256	128	32	16	8	2

Note that in each column $xy = 64$

y	4	6	—1	—$\frac{1}{2}$
x	3	2	—12	—24

Find the relation between y and x in each column.

$xy = 12.$

In the above two examples we can see that the product of x and y does not change.

If the product of two variables does not change, then that function is called an Inverse Variation function.

$x\,y = k$ or $y = \frac{k}{x}$; $f(x) = \frac{k}{x}$

Further as x increases, y decreases. Take the ratio of any two values of k, say 2 and 32. It is 2 : 32. Find the ratio of the corresponding values of y. It is 32 : 2. We find the ratio in the range set is inversed. Such functions are called inverse variation functions.

Example:

Is $y = 5x$ an inverse variation function ?

x	y	xy
2	10	20
4	20	80
—1	—5	5

Ratio between 2 elements of x is 2 : 4 = 1 : 2

Ratio between corresponding elements of y is 10 : 20 = 1 : 2.

∴ $y = 5x$ is not an inverse variate function, but a direct variate function.

Exercise 5—6

1. Find which of the following tables represent inverse variation function.

(1)

x	y
3	15
5	9
15	3

(2)

c	d
—2	3
—1	- 6
— 3	2

(3)

x	y
1	1
3/2	2/3
4/5	5/4

2. If $x = 3$; $y = 12$ find y when $x = 6$, if $y = f(x)$ is an inverse variate function.

3. If $x = \frac{1}{9}$; $y = 81$ find x when $y = 3$, if y is an inverse variate function of x.

Answers

1. (1), (3) are Inverse variation functions.

2. 6 3. 3

5—7. Linear function

Consider the equation $y = 3x + 5$; $x, y \in \mathbf{R}$

Does the value of y change according to the value of x?

x	—1	1	2	3	4
$y = f(x)$	2	8	11	14	17

If the value of x is multiplied by 3 and 5 is added, we will get y.

It is written symbolically in the form $f(x) = ax + b$.

Its graph is a line segment.

This is neither a horizontal line nor a vertical line.

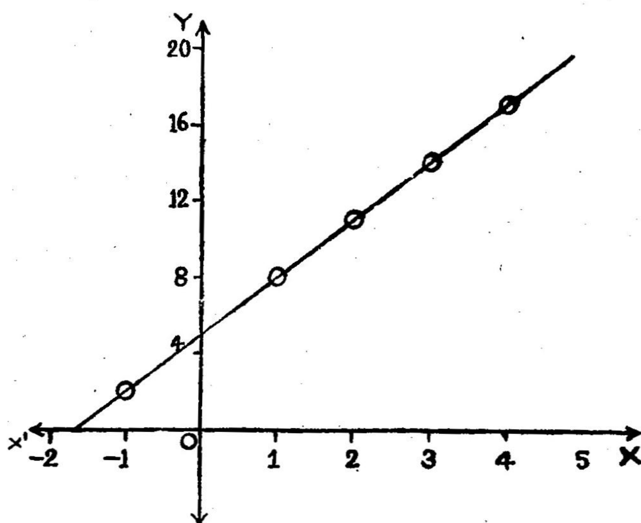


Fig. 1-39.

MATHEMATICS CLUB — ACTIVITY I

Verification of arithmetical operations

When you do problems in arithmetic, involving the four operations, you have to know that you have not made any mistake. Here below are given two tests which will help you to minimise the possibilities of mistakes.

Test 1

1. Add the digits in the numbers given until they sum upto a single digit.

2. Do the given operation on the values thus obtained and convert the same into a single digit.

3. Convert the number obtained by the operation on the given numbers into a single digit.

4. If you get the same number in steps 2 and 3, your answer may not be wrong.

Exomple 1

371 × 43 = 15953

371 = 3 + 7 + 1 = 11; 1 + 1 = 2

43 = 4 + 3 = 7

2 × 7 = 14; 1 + 4 = 5

15953 = 1 + 5 + 9 + 5 + 3 = 23; 2 + 3 = 5.

Test 2

1. Find the sum of the numbers in the odd digit places starting from the unit's place. Find the sum of the numbers in the even digit places. Find the difference.

2. Do the operation on the numbers obtained.

3. Verify.

4. If the difference of the sums is negative, take the 11 complement. (e. g.) — 3 should be taken as 11 — 3 = 8.

Example 2

557 × 25 = 13925

557 = (7 + 5) — 5 = 7

25 = 5 — 2 = 3

10
7 3
10

7 × 3 = 21; 1 — 2 = — 1; 11 — 1 = 10

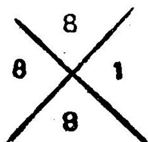
13925 = (5 + 9 + 1) — (2 + 3) = 15 — 5 = 10

If both the tests hold good, the chance of error is less.

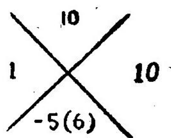
Example 3

485 × 208 = 13580

Test 1



Test 2



Test 1 holds good; Test 2 does not hold good. Some error has crept in the manipulation.

Verify

1. 4505 × 381 = 1754505
2. 7285 × 437 = 3184645
3. 647 × 872 = 564184
4. 389 × 211 = 82979

2. NUMBER SYSTEM

2—1. Number System — Numeration

We know that the numbers that we are using are in base ten system. In this system, any number can be written with the numerals 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. There are two values for each numeral. One is actual value and the other is the place value.

A digit in a number has place value according to its place.

Example :

The number 5 3 2 3 has two threes. The actual value is 3 for both. But the place value of the first 3 is 3 since it is in the unit's place and the place value of the other 3 is 300 (3 × 100) since it is in the 100s place.

The place value increases as powers of 10 (1, 10, 100,...) from right to left.

10^4	10^3	10^2	10^1	10^0
Ten thousands	Thousands	Hundreds	Tens	Units
4	6	2	5	8

In 46258,

8 is in the units place. (This is 10^0) = 8×10^0 = 8

5 is in the tens place. (This is 10^1) = 5×10^1 = 50

2 is in the hundreds place. (This is 10^2) = 2×10^2 = 200

6 is in the thousands place. (This is 10^3) = 6×10^3 = 6000

4 is in the ten thousands place (This is 10^4) = 4×10^4 = 40000

46258

Exercise 1—1

1. Write down the following numbers as powers of 10.

Example :

[467 = $(4 \times 10^2) + (6 \times 10) + 7 \times 1$]

(i) 384 (ii) 6840 (iii) 54698 (iv) 108439 (v) 16705

2. Find the relation between the number of zeros in 10, 100, 1000, ... and the powers of 10.

1—2. Base five system — Quenary System

We are using 10 numerals in base ten system, otherwise called denary system.

We group by tens in base ten system. 10 units make 1 ten; 10 tens make 1 hundred or 10^2s. 10 hundreds make 1 thousand or 10^3s. Similarly we group by fives in base five number system. 0, 1, 2, 3, 4 are the only five numerals in base 5 number system.

Let us take fifteen apples and find out their numerals in base 10 and in base 5.

In base 10 system,

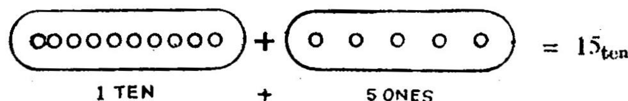


Fig. 2-1

In base 5 system,

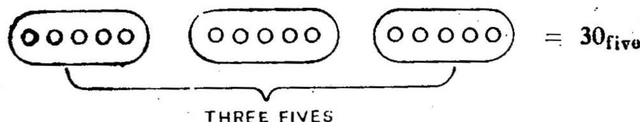


Fig. 2–2

This is written as $(30)_{five}$. In base 5 number system, we group by 5's The place values increase in powers of 5 i.e. 5^0, 5^1, 5^2, 5^3, ... from the right to the left.

$(8)_{10}$ = (O O O O O) + (O O O)

1 FIVE + 3 ONES

Fig. 2-3.

This is written as $(13)_{five}$

Note **: To denote the numbers in base five system, we use 5 as suffix, that is 13_5. This denotes a base five number which is read as 'one three base five'**

$(14)_{ten}$ = (OOOOO) + (OOOOO) + (OOOO)

2 FIVES + 4 ONES

Fig. 2-4.

$(14)_{ten} = (24)_{five}$ This is read as 'two four base five'.

DENARY NUMERALS		BASE 5 NUMERALS	
1	O	1	ONE
2	OO	2	TWO
3	OOO	3	THREE
4	OOOO	4	FOUR
5	OOOOO	10_5	ONE FIVE + O UNITS
6	OOOOOO	11_5	ONE FIVE + 1 UNIT
7	OOOOOOO	12_5	ONE FIVE + 2 UNITS
8	OOOOOOOO	13_5	ONE FIVE + 3 UNITS
9	OOOOOOOOO	14_5	ONE FIVE + 4 UNITS
10	OOOOOOOOOO	20_5	TWO FIVES + O UNITS

Note that there are no numerals as 5, 6, 7, 8, 9 in base 5 number system.

The number (58) contains 11 fives and 3 ones.

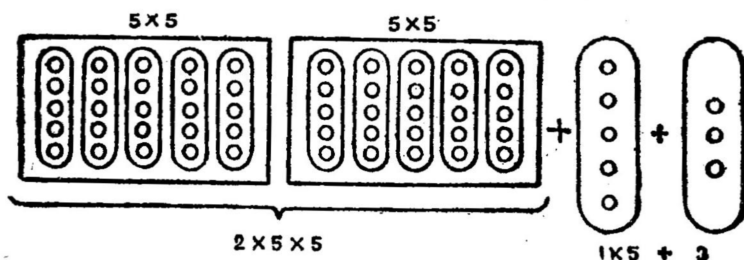


Fig. 2-5.

$58 = 2\times5^2 + 1\times5 + 3.$

This can be written as 213_5

5^4 =625	5^3 =125	5^2 =25	5^1 =5	5^0 =1
		2	1	3

→ $3\times1 = 3$

→ $1\times5 = 5$

→ $2\times25 = 50$

$\overline{58}$

To convert $(58)_{10}$ to base five system we use division method also.

$$\begin{array}{r|l} 5 & 58 \\ 5 & 11 - 3 \\ & 2 - 1 \end{array} \qquad 58_{ten} = (213)_{five}$$

To write $(412)_{ten}$ in base five system:

$$\begin{array}{r|l} 5 & 412 \\ 5 & 82 - 2 \\ 5 & 16 - 2 \\ & 3 - 1 \end{array} \qquad 412_{ten} = (3122)_{five}$$

Exercise 1—2

1. The place values in base five system are — of 5.

2. Convert the following denary numbers to base five numbers.

(i) 66 (ii) 143 (iii) 403 (iv) 210 (v) 300.

3. Convert the following base five numbers into denary numbers.

(i) 13_5 (ii) 40_5 (iii) 222_5 (iv) 123_5 (v) 403_5

(vi) 2134_5

4. The number next to 4 in base five system is 10_5. Write the numbers next to (i) 444_5 (ii) 404_5 (iii) 3444_5.

5. Which of the following base five numbers are even numbers ?

(i) 4_5 (ii) 11_5 (iii) 14_5 (iv) 23_5 (v) 22_5

(vi) 32_5 (vii) 111_5 (viii) 123_5 (ix) 100_5

Answers

1. Powers

2. (i) 231_5 (ii) 1033_5 (iii) 3103_5 (iv) 1320_5 (v) 2200_5

3. (i) 8 (ii) 20 (iii) 62 (iv) 38 (v) 103 (vi) 294

4. (i) 1000_5 (ii) 410_5 (iii) 4000_5

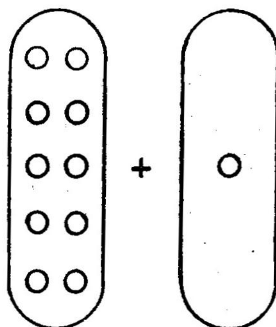
5. (i), (ii), (v), (viii) are even numbers.

1—3. Binary Number System

0, 1, 2, 3, 4, 5, 6, 7, 8, 9 are numerals used in denary number system and 0, 1, 2, 3, 4 are the numerals used in base five number system. Similarly we are using the numerals 0 and 1 in base two system. We group the numbers two by two in base two number system.

The place values in base 10 system are 10^0, 10^1, 10^2, 10^3, ... from the right to the left, and in base 5 system, they are 5^0, 5^1, 5^2, 5^3, ... from the right to the left. Similarly the place values in base two system will be 2^0, 2^1, 2^2, 2^3, ... from the right to the left.

$(11)_{10}$ = 1 ten + 1 one

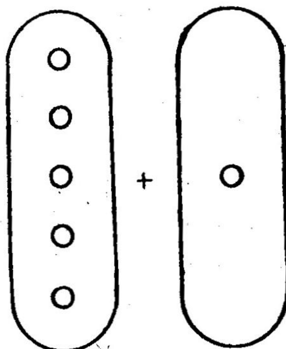


1 TEN + 1 ONE

Fig. 2–6.

$(6)_{10}$ = 1 five + 1 one

This is written as $(11)_5$.



1 FIVE + 1 ONE

Fig. 2–7.

$(3)_{10}$ = 1 two + 1 one

This is written as $(11)_2$.

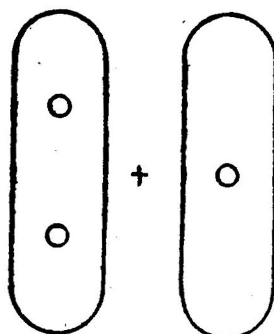


Fig. 2-8.

Note that the place value in Binary Number system is a power of 2.

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
128	64	32	16	8	4	2	1

Denary System	1	2	3	4	5	6	7	8	9	10
Binary System	1	10	11	100	101	110	111	1000	1001	1010

We can use this table to convert $(18)_{10}$ to base two system.

$$18 = 16+2$$
$$= 1\times2^4 + 1\times2^1.$$

$\therefore$ $(18)_{10}$ is written as 10010 in base two system.

Division method:

$$\begin{array}{r|l} 2 & 18 \\ 2 & 9-0 \\ 2 & 4-1 \\ 2 & 2-0 \\ & 1-0 \end{array}$$

$(18)_{10} = (10010)_2$

In Denary System

$$\begin{array}{l} 1\ 0\ 0\ 1\ 0 \\ 0 \times 2^0 = 0 \\ 1 \times 2^1 = 2 \\ 0 \times 2^2 = 0 \\ 0 \times 2^3 = 0 \\ 1 \times 2^4 = 16 \\ \hline \qquad\qquad 18 \end{array}$$

i.e. $(10010)_2 = (18)_{10}$

Exercise 1—3

1. Convert the following denary numbers into base two numbers.

(i) 19 (ii) 14 (iii) 27 (iv) 30

2. Rewrite the following binary numbers as denary numbers.

(i) 101_2 (ii) 1101_2 (iii) 11111_2
(iv) 100001_2 (v) 110010_2

3. Write down two Successors of the following:

(i) 11_2 (ii) 110_2 (iii) 1001_2 (iv) 1100_2
(v) 1_2 (vi) 10_2 (vii) 111_2 (viii) 1010_2

Answers

1. (i) 10011_2 (ii) 1110_2 (iii) 11011_2 (iv) 11110_2

2. (i) 5 (ii) 13 (iii) 31 (iv) 33 (v) 50

3. (i) 100_2, 101_2 (ii) 111_2, 1000_2 (iii) 1010_2, 1011_2
(iv) 1101_2, 1110_2 (v) 10_2, 11_2 (vi) 11_2, 100_2
(vii) 1000_2, 1001_2 (viii) 1011_2, 1100_2

2—1. Addition of Binary numbers

Addition in binary numbers can be done as in denary numbers.

Look at the table.

+	0	1
0	0	1
	0	10

Examples :

(1) 1000_2 + 101_2 = 1101_2

(2) 11_2 + 101_2 = 1000_2

(3) 111101_2 + 10010_2 = 1001111_2

Exercise 2—1

Add the following binary numbers.

(1) $10110_2 + 1101_2$ (2) $1101_2 + 101_2$

(3) $11111_2 + 10001_2$ (4) $100101_2 + 111011_2$

(5) $111_2 + 111_2$

Answers

(1) 100011_2 (2) 10010_2 (3) 110000_2 (4) 1100000_2

(5) 10110_2

2—2. Multiplication in Binary System

We can frame the multiplication table in a similar way to that of addition table.

×	0	1
0	0	0
1	0	1

Multipication in Binary system can be done as in denary system.

Example : $111_2 \times 11_2$

111_2
11_2
111
111
10101

(2) $101_2 \times 10_2$

101_2
10_2
1010_2

Exercise 2—2

Multiply the following binary numbers:

(1) $101_2 \times 11_2$ (2) $110_2 \times 101_2$ (3) $1010_2 \times 110_2$

(4) $1101_2 \times 101_2$ (5) $1100_2 \times 101_2$

Answers

(1) 1111_2 (2) 11110_2 (3) 111100_2 (4) 1000001_2

(5) 111100_2

3. Clock Numbers

The school begins at 10 in the morning and closes after 6 hours. What is the time then ?

We know that it is 4 in the evening. The numbers arranging from 1 to 12 repeat often in the clock. Now we can discuss the number system which has such repeating numbers.

The days of a week repeat themselves. If we begin with Sunday as 1 and end with Saturday as 7, these numbers are also having the pattern of the clock numbers. The rotation in clock numbers is 12 and the rotation in days of a week is 7.

Similarly we can form different clock numbers with different rotations

The addition and multiplication tables for clock number 7 are given on the next page.

+	1	2	3	4	5	6	7
1	2	3	4	5	6	7	1
2	3	4	5	6	7	1	2
3	4	5	6	7	1	2	3
4	5	6	7	1	2	3	4
5	6	7	1	2	3	4	5
6	7	1	2	3	4	5	6
7	1	2	3	4	5	6	7

×	1	2	3	4	5	6	7
1	1	2	3	4	5	6	7
2	2	4	6	1	3	5	7
3	3	6	2	5	1	4	7
4	4	1	5	2	6	3	7
5	5	3	1	6	4	2	7
6	6	5	4	3	2	1	7
7	7	7	7	7	7	7	7

The addition table for clock number 12:

+	1	2	3	4	5	6	7	8	9	10	11	12
1	2	3	4	5	6	7	8	9	10	11	12	1
2	3	4	5	6	7	8	9	10	11	12	1	2
3	4	5	6	7	8	9	10	11	12	1	2	3
4	5	6	7	8	9	10	11	12	1	2	3	4
5	6	7	8	9	10	11	12	1	2	3	4	5
6	7	8	9	10	11	12	1	2	3	4	5	6
7	8	9	10	11	12	1	2	3	4	5	6	7
8	9	10	11	12	1	2	3	4	5	6	7	8
9	10	11	12	1	2	3	4	5	6	7	8	9
10	11	12	1	2	3	4	5	6	7	8	9	10
11	12	1	2	3	4	5	6	7	8	9	10	11
12	1	2	3	4	5	6	7	8	9	10	11	12

The multiplication table for clock number 12:

×	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	2	4	6	8	10	12
3	3	6	9	12	3	6	9	12	3	6	9	12
4	4	8	12	4	8	12	4	8	12	4	8	12
5	5	10	3	8	1	6	11	4	9	2	7	12
6	6	12	6	12	6	12	6	12	6	2	6	12
7	7	2	9	4	11	6	1	8	3	10	5	12
8	8	4	12	8	4	12	8	4	12	8	4	12
9	9	6	3	12	9	6	3	12	9	6	3	12
10	10	8	6	4	2	12	10	8	6	4	2	12
11	11	10	9	8	7	6	5	4	3	2	1	12
12	12	12	12	12	12	12	12	12	12	12	12	12

Exercise 3

1. If 2nd August is Sunday, what is the day of 18th August ?

2. If 1st September is Monday, find the day of 22nd September.

Answers

1. Tuesday 2. Monday

4—1. Modular Numbers

We have learnt about the clock numbers in the previous lesson. Let us now try to find a new pattern of numbers different from the clock numbers.

Look at the table given below. What can be inferred about the remainders when a whole number is divided by 4 ?

Dividend	Remainder when it is divided by 4	Dividend	Remainder when it is divided by 4
0	0	6	2
1	1	7	3
2	2	8	0
3	3	9	1
4	0	10	2
5	1	11	3
		12	0

We will get 0, 1, 2, 3 as remainders when any whole number is divided by 4.

In general, the remainders when any whole number is divided by m are 0, 1, 2, 3, ... (m−1).

The infinite set of whole numbers {0, 1, 2, 3, ...} when divided by 4 give remainders which form a finite set {0, 1, 2, 3}.

The remainder when 18 is divided by 4 is 2.

This is written as $18 \equiv 2 \pmod 4$

$$2 \equiv 2 \pmod 4$$

$$24 \equiv 4 \pmod 5$$

$$34 \equiv 4 \pmod 5$$

$$\therefore \quad 24 \equiv 34 \pmod 5$$

We can see that all numbers which are having the same remainder when divided by 5 are identical.

Modulus — Addition

(1) $37 \equiv 2 \pmod 5$

$$27 \equiv 2 \pmod 5$$

$$\therefore \quad 37 \equiv 27 \pmod 5$$

$$37+27 \equiv 2+2 = 4 \pmod 5$$

(2) $49 \equiv 4 \pmod 5$

$$34 \equiv 4 \pmod 5$$

$$49+34 \equiv ? \pmod 5$$

$$4+4 \equiv 3 \pmod 5$$

$\therefore \quad 49+34 \equiv 3 \pmod 5$. **Verify the answer.**

Modulus — Multiplication

(1) $14 \equiv 2 \pmod 3$

$$8 \equiv 2 \pmod 3$$

$$14 \times 8 \equiv ? \pmod 3$$

$$2 \times 2 \equiv 1 \pmod 3$$

$$\therefore \quad 14 \times 8 \equiv 1 \pmod 3$$

(2) $10 \equiv 1 \pmod{3}$

$7 \equiv 1 \pmod{3}$

$\therefore \quad 10 \times 7 \equiv 1 \pmod{3}$

$14 \times 10 \equiv 8 \times 7 \equiv 2 \pmod{3}$ is also true.

Examples:

1. $24 \equiv 0 \pmod{6}$

$17 \equiv 5 \pmod{6}$

$24 \times 17 \equiv ? \pmod{6}$

$24 \times 17 \equiv 0 \pmod{6} \qquad [0 \times 5 = 0]$

2. Show that $2^{12} \equiv 6 \pmod{10}$

$2^4 = 16$

$16 \equiv 6 \pmod{10}$

$2^{12} = 2^4 \times 2^4 \times 2^4$

$\therefore \quad 2^4 \times 2^4 \times 2^4 \equiv 6 \times 6 \times 6 \pmod{10}$

$6 \times 6 \times 6 \equiv 6 \times 6 \pmod{10}$

$6 \times 6 \equiv 6 \pmod{10}$

$\therefore \quad 2^{12} \equiv 6 \pmod{0}$

Aliter

$2^6 = 64$

$64 \equiv 4 \pmod{10}$

$2^{12} = 2^6 \times 2^6$

$\therefore \quad 2^6 \times 2^6 \equiv 4 \times 4 \pmod{10}$

$\therefore \quad 2^{12} \equiv 6 \pmod{10}$

3. Show that $5^n \equiv 5 \pmod{10}$

$5 \equiv 5 \pmod{10}$

$5^2 \equiv 5 \pmod{10}$

$5^6 \equiv 5 \pmod{10}$

We know that the power of 5 ends with 5.

$\therefore \quad 5^n \equiv 5 \pmod{10}$

4. $9^5 \equiv ?\ (\text{mod } 10)$

$9 \equiv 9\ (\text{mod } 10)$

$9^2 \equiv 1\ (\text{mod } 10)$

$9^3 \equiv 9\ (\text{mod } 10)$ $\quad 9^3 = 729$

$9^4 \equiv 1\ (\text{mod } 10)$

$9^5 \equiv 9\ (\text{mod } 10)$

Can you generalise this result to find $9^n \equiv ?\ (\text{mod } 10)$

Exercise 4—1

Find the following:

(1) $4^9 \equiv \quad (\text{mod } 5)$ (2) $2^{20} \equiv \quad (\text{mod } 5)$

(3) $3^{12} \equiv \quad (\text{mod } 6)$ (4) $9^{27} \equiv \quad (\text{mod } 10)$

(5) $9^{18} \equiv \quad (\text{mod } 10)$ (6) $5^{81} \equiv \quad (\text{mod } 5)$

(7) $14^5 \equiv \quad (\text{mod } 9)$

(8) Find the least value of x when $x+3 \equiv 2\ (\text{mod } 5)$; $x \in N$.

(9) Find the least value of x when $x^2+4 \equiv 4\ (\text{mod } 7)$; $x \in N$

(10) $10^{11} \equiv \quad (\text{mod } 11)$

Answers

(1) 4 (2) 1 (3) 3 (4) 9 (5) 1

(6) 5 (7) 2 (8) 5 (9) 7 (10) 10

4—2. Modulus — Multiplication — Use of table

Look at the multiplication table of mod 4. (Why are we using only the numbers 0, 1, 2, 3 ?)

×	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	2
3	0	3	2	1

From the table compute the following:

$$27 \equiv \quad (\text{mod } 4)$$

$$38 \equiv \quad (\text{mod } 4)$$

$$27 \times 38 \equiv \quad (\text{mod } 4)$$

$$27 \equiv 3 \ (\text{mod } 4)$$

$$38 \equiv 2 \ (\text{mod } 4)$$

$$27 \times 38 \equiv 3 \times 2 \ (\text{mod } 4)$$

$$\therefore \ 27 \times 38 \equiv 2 \ (\text{mod } 4)$$

Exercise 4—2

1. Construct the multiplication table for mod 5 and compute the following:

(a) $38 \equiv ? \ (\text{mod } 5)$ (b) $49 \equiv ? \ (\text{mod } 5)$

(c) $38 \times 49 \equiv ? \ (\text{mod } 5)$ (d) $16 \times 25 \equiv ? \ (\text{mod } 5)$

2. $34 + 68 + 43 \equiv ? \ (\text{mod } 10)$

3. $2^6 \equiv \quad (\text{mod } 9)$

4. Find the least value of x when

$(x+1)(x+2) \equiv 2 \ (\text{mod } 5); \ x \in N.$

5. Find the day of the 37th day from today if today is Wednesday.

6. Find the 30th month from this month if this month is October.

Answers

1. (a) 3 (b) 4 (c) 2 (d) 0

2. 5 3. 1 4. 2 5. Thursday 6. March

4—3. Modulus — Division

The multiplication table for mod 6 is given below.

×	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	2	3	4	5
2	0	2	4	0	2	4
3	0	3	0	3	0	3
4	0	4	2	0	4	2
5	0	5	4	3	2	1

From this table we can say that

$$2 \times 3 \equiv 0 \pmod{6}$$

In ordinary division we know that

$$2 \times 3 = 6$$

$$2 = \frac{6}{3}; \qquad 3 = \frac{6}{2}$$

Let us verify this in modular Arithmetic.

Note that $2 \times 3 = 0$

$$\therefore \quad \frac{0}{2} = 3; \quad \frac{0}{3} = 2.$$

We know that if zero is divided by any number, then the answer will be 0 in ordinary division. But it is not so in modular Arithmetic.

$$4 \times 3 \equiv 0 \pmod 6$$

$$\therefore \frac{0}{3} = 4 ; \frac{0}{4} = 3 \text{ are true.}$$

3 and 4 are called zero divisions in modulo 6. Find the zero divisions from the table.

5. Decimal Fraction — Revision

$$\frac{1}{10} = \cdot 1$$

$$\frac{1}{5} = \cdot 2$$

$$\frac{37}{100} = \cdot 37$$

$$\frac{7}{20} = \cdot 35$$

You have learnt that these fractions are terminating decimal fractions. Give some examples for terminating decimals.

$\frac{1}{3} = \cdot 33 \ldots$ This is a non-terminating decimal. This can be written as $\cdot\dot{3}$. This is a recurring decimal.

$\frac{1}{11} = \cdot 090909 \ldots$ Here $\cdot 09$ is a periodic decimal. This can be written as $\cdot 09\,\dot{0}\dot{9}$. This is also a non-terminating recurring decimal.

You know that terminating, non-terminating and recurring decimals are rational numbers.

Exercise 5

1. Rewrite the following rational numbers as recurring decimals.

(a) $\frac{1}{9}$ (b) $\frac{23}{45}$ (c) $\frac{7}{15}$ (d) $\frac{25}{37}$ (e) $\frac{5}{6}$

(f) $\frac{32}{111}$ (g) $\frac{1}{13}$

2. Rewrite the following rational numbers as decimals (correct to four places).

(a) $\frac{1}{11}$ (b) $\frac{2}{11}$ (c) $\frac{3}{11}$ (d) $\frac{9}{11}$ (e) $\frac{14}{11}$

(f) $\frac{23}{11}$

3. (a) Is there any relation between the decimal numerals of $\frac{1}{11}$ and $\frac{2}{11}$?

(b) Is there any relation between the decimal numeral of $\frac{1}{11}$ and the decimal numerals of $\frac{3}{11}$, $\frac{9}{11}$, $\frac{14}{11}$ and $\frac{23}{11}$?

4. Can you convert $\frac{5}{11}$ as a decimal fraction without doing the division ?

5. $\frac{4}{11}$ is 4 times $\frac{1}{11}$. Is this true ?

Answers

1. (a) $\cdot\dot{1}$ (b) $\cdot 5\dot{1}$ (c) $\cdot 4\dot{6}$ (d) $\cdot\dot{6}\dot{7}\dot{5}$

2. (a) ·0909 (b) ·1818 (c) ·2727

6. Irrational numbers

We have seen that all rational numbers can be written either as a terminating or a non-terminating decimal.

Look at the number 2·4035789201 ... This is neither a terminating nor a recurring decimal.

2·444 .... This is a rational number.

$$2{\cdot}444\ldots\ldots = \frac{22}{9}$$

2·444 ... ↑ Recurring decimal ↓ This can be written as $2{\cdot}4\dot{4}$

2·4343343334 ... ↑ This is a non-recurring non-terminating decimal ↓ This cannot be written with a special notation like recurring decimal.

Non-terminating, non-recurring decimals are known as **Irrational numbers.**

Exercise 6

1. Say whether the following numbers are rational or irrational numbers.

(1)	7·323232...	(2)	3·6262262226...	(3)	$\cdot 34\dot{1}4\dot{1}$
(4)	$\sqrt{9}$	(5)	$\sqrt{10}$	(6)	$-\sqrt{16}$
(7)	$-3\frac{1}{2}$	(8)	·49505[illegible]52 ...	(9)	·027
(10)	·441441144111....				

Answers

(1), (3), (4), (6), (7), (9) — Rational numbers

(2), (5), (8), (10) — Irrational numbers.

7. Some irrational numbers

We have seen that any rational number can be written as either a terminating decimal or a non-terminating recurring decimal.

What is the length of the side of a square of area 16 cm^2 ? 4 cm ($\sqrt{16}$)

The side of a square of area 2·56 cm^2 is 1·6 cm. But can you tell the length of the side of a square of area 2 cm^2 ?

This is $\sqrt{2}$ cm.

Express $\sqrt{2}$ as a decimal.

$\sqrt{2} = 1{\cdot}41421 \ldots$
This is a non-terminating, non-recurring decimal. So this is an irrational number.

$\sqrt{2} \simeq 1{\cdot}4142$ (correct to 4 places)

	1·41421
	2·0000000000
1	1
	100
24	96
	400
281	281
	11900
2824	11296
	60400
28282	56564
	383600
282841	282841
	100759

The difference between 2 and the square of 1.4142 is

$$2 - 1{\cdot}99996164$$

$$= {\cdot}00003836.$$

Using the same procedure we can find the approximate value of $\sqrt{3}$.

$$\sqrt{3} \simeq 1{\cdot}7321 \text{ (correct to 4 places)}$$

More accuracy will be obtained if we find it for more places.

π :

You know that the circumference of a circle is π times its diameter (πd).

π is an irrational number. The approximate values of π are $\frac{22}{7}$, 3·14, 3·1416, $\frac{355}{113}$

These values are not the actual values.

$\sqrt{2}$, $\sqrt{3}$ in number line

Using the number line draw the two axes as in Fig. 2-9.

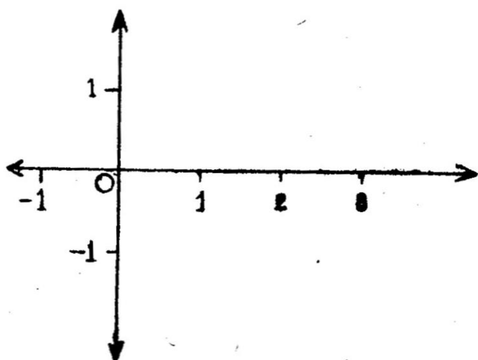


Fig. 2-9

Join the line segment AB where the co-ordinates of A are (1, 0) and the co-ordinates of B are (0, 1).

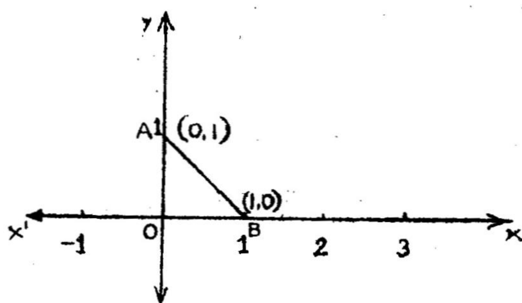


Fig. 2-10

By Pythagoras theorem, the square on the hypotenuse of a right angled triangle is equal to the sum of the squares on the other two sides.

$$AB^2 = OA^2 + OB^2$$

$$AB^2 = 1^2 + 1^2 = 2$$

$$AB = \sqrt{2}$$

Mark the length of the line segment $\overline{AB}$ on the number iine with the help of compasses. This point denotes $\sqrt{2}$ in the number line.

Similarly we can plot the point which denotes $\sqrt{3}$ in the number line.

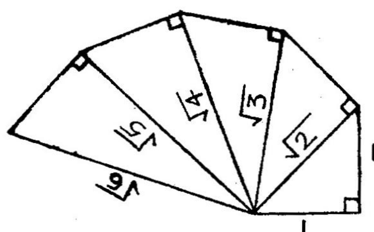


Fig. 2-11

Look at the figure.

We can make use of the above figure to find $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{6}$, ...

8—1. Set of Real Numbers and its subsets

We have learnt so far about many sets of numbers.

What do the letters N, W, Z, Q denote? We know that $N \subset W \subset Z \subset Q$ is true.

We have seen some numbers which do not belong to the above sets and we know that those numbers are irrational numbers. From this we can conclude that all numbers are either rational or irrational numbers.

The set of rational and irrational numbers is known as the set of Real numbers and it is denoted as $\mathbb{R}$.

Look at the figure. What do we note from this?

N, W, Z and Q are the subsets of the set of Real numbers.

$N \subset W \subset Z \subset Q \subset \mathbf{R}$

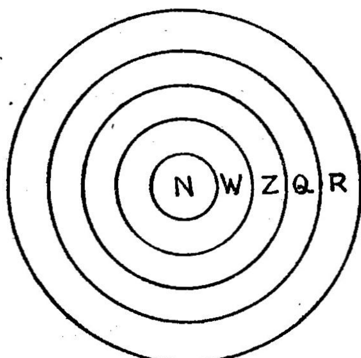


Fig. 2-12

When we note the rational numbers on a number line, we have some gaps in between. Why is this ?

All numbers cannot be rational numbers. There are some irrational numbers like $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, ... in the number line. Rational and irrational numbers complete the number line.

Properties of Number System:

The irrational numbers possess the following properties.

Addition	Multiplication
Closure with respect to addition	Closure with respect to multiplication
Commutative property $a+b = b+a$	Commutative property $a \times b = b \times a$
Associative property $(a+b)+c = a+(b+c)$	Associative property $(a \times b) \times c = a \times (b \times c)$
Additive identity $a + 0 = 0 + a = a$	Multiplicative identity $a \cdot 1 = 1 \cdot a = 0$
Additive inverse $a+(-a) = (-a)+a = 0$	Multiplicative inverse $a \times \frac{1}{a} = \frac{1}{a} \times a = 1$ where $a \neq 0$
Distributive property $a(b+c) = ab+ac$	
	Multiplicative property of 0 $a \times 0 = 0 \times a =$ zero

Order Property:

Take any two numbers a and b. Any one of the three statements will be true.

$$a < b;\quad a = b;\quad a > b.$$

The set of real numbers possess all the properties of the rational numbers. Why? Above all these, the set of real numbers possess the density property.

Density property :

Every point of a number line corresponds to a real number. (All numbers are real numbers.)

We have seen many number systems so far. All these number systems possess the following properties:

1. Closure property
2. Commutative property
3. Associative property
4. Distributive property
5. Order property
6. Identity

Properties	Natural numbers	Whole numbers	Integers	Positive rational	Rational numbers	Real numbers
Additive property		✓	✓	✓	✓	✓
Multiplicative inverse				✓	✓	✓
Additive inverse			✓		✓	✓
Multiplicative property of 0		✓	✓	✓	✓	✓

9—1. Square Root

Revision

16 is the square of 4

4 is the square root of 16

16 is the square of —4

—4 is the square root of 16

The square roots of 16 are 4 and —4.

We know that $a^2 = (a)^2$ or $(-a)^2$

For any whole number, there will be two square roots, one a positive integer and the other a negative integer.

The square roots of 25 are 5 and —5. We will consider the positive roots of whole numbers only in this lesson.

(1) $441 = 3 \times 3 \times 7 \times 7$

$= 3^2 \times 7^2$

$\sqrt{441} = \sqrt{3^2 \times 7^2} = 3 \times 7 = 21$

(2) $1764 = \sqrt{2^2 \times 3^2 \times 7^2}$

$= 2 \times 3 \times 7$

$= 42$

(3) $\sqrt{27 \times 15 \times 20} = \sqrt{3^3 \times 3 \times 5 \times 5 \times 2^2}$

$= \sqrt{3^4 \times 5^2 \times 2^2}$

$= 3^2 \times 5 \times 2$

$= 90.$

Exercise 9—1

1. Find the square root by prime factorisation :

(a) 2304 (b) 7056 (c) 39204 (d) 50625

(e) 44100 (f) 26244.

2. **Find the square root of the following products:**

(a) 18 × 32 (b) 44 × 99 (c) 50 × 72

(d) 72 × 128 (e) 108 × 147 (f) 112 × 175

(g) 15 × 35 × 21 (h) 30 × 35 × 42

(i) 21 × 56 × 96 (j) 18 × 21 × 30 × 35

3. **Find x.**

(a) $6x^2 = 486$ (b) $12x^2 = 432$ (c) $60x^2 = 2940$

(d) $2\frac{1}{2}x^2 = 640$ (e) $\frac{1}{2}x^2 = 128$ (f) $\frac{1}{3}x^2 = 147$

(g) $x = \sqrt{a^6}$ (h) $x = \sqrt{9a^8}$

4. **Find the value of 'r' in $A = \pi r^2$ if $A = 616$, $\pi = 22/7$**

Answers

1. (a) 48 (b) 84 (c) 198 (d) 225 (e) 210
(f) 162

2. (a) 24 (b) 66 (c) 60 (d) 96 (e) 126 (f) 140
(g) 105 (h) 210 (i) 336 (j) 630

3. (a) 9 (b) 6 (c) 7 (d) 16 (e) 16 (f) 21
(g) a^3 (h) $3a^4$

4. 14

9—2. Square root of fractional numbers

$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$$\text{Square of } \frac{1}{2} = \left(\frac{1}{2}\right)^2 = \frac{1^2}{2^2} = \frac{1}{4}$$

$$\therefore \text{ Square root of } \frac{1}{4} \text{ is } \frac{1}{2}$$

$$\sqrt{\frac{1}{4}} = \frac{\sqrt{1}}{\sqrt{4}} = \frac{1}{2}$$

$$\frac{2}{5} \times \frac{2}{5} = \frac{4}{25}$$

Square root of $\frac{4}{25}$ is $\sqrt{\frac{4}{25}} = \frac{\sqrt{4}}{\sqrt{25}} = \frac{2}{5}$

We can follow the above procedure if the numerator and denominator are square numbers.

Square of $\frac{9}{7}$ is $\frac{9}{7} \times \frac{9}{7} = \frac{81}{49} = 1\frac{32}{49}$

$1\frac{32}{49}$ is a mixed number.

$$\sqrt{1\frac{32}{49}} = \sqrt{\frac{81}{49}} = \frac{\sqrt{81}}{\sqrt{49}} = \frac{9}{7} = 1\frac{2}{7}$$

Square root of a fraction is $\frac{\text{Square root of Numerator}}{\text{Square root of Denominator}}$

Examples

1. $\sqrt{\frac{16}{25}} = \frac{\sqrt{16}}{\sqrt{25}} = \frac{4}{5}$

2. $\sqrt{4\frac{21}{25}} = \sqrt{\frac{121}{25}} = \frac{\sqrt{121}}{\sqrt{25}} = \frac{11}{5} = 2\frac{1}{5}$

3. $\sqrt{\frac{63}{112}} = \sqrt{\frac{63 \div 7}{112 \div 7}} = \sqrt{\frac{9}{16}} = \frac{3}{4}$

[7 is a common factor to both the numerator and denominator.]

Note:

1. If a number is greater than unity, then its square root is less than that number; $\sqrt{16} = 4$.

2. Square root of unity is 1; $\sqrt{1} = 1$.

3. If a number is less than unity, then the square root of that number will be greater than that number.

$\sqrt{\frac{1}{4}} = \frac{1}{2}$. Why?

Exercise 9—2

Find the square root of the following fractions:

1. (a) $\frac{49}{64}$ (b) $\frac{25}{36}$ (c) $\frac{64}{81}$ (d) $\frac{36}{49}$

2. (a) $1\frac{9}{16}$ (b) $2\frac{14}{25}$ (c) $1\frac{7}{9}$ (d) $1\frac{13}{36}$

3. (a) $\frac{720}{245}$ (b) $\frac{392}{512}$ (c) $\frac{175}{28}$ (d) $\frac{360}{810}$

4. Find the value of $\sqrt{768}$ if $\sqrt{3} \simeq 1{\cdot}732$

Answers

1. (a) $\frac{7}{8}$ (b) $\frac{5}{6}$ (c) $\frac{8}{9}$ (d) $\frac{6}{7}$

2. (a) $1\frac{1}{4}$ (b) $1\frac{3}{5}$ (c) $1\frac{1}{3}$ (d) $1\frac{1}{6}$

3. (a) $1\frac{5}{7}$ (b) $\frac{7}{8}$ (c) $2\frac{1}{2}$ (d) $\frac{2}{3}$

4. 27·712

9—3. Square root — Division Method

1. $2^2 = 4$; $7^2 = 49$; $9^2 = 81$

How many digits will there be in the square of a single digit ?

2. $30^2 = 900$; $35^2 = 1225$; $90^2 = 8100$

Will there be 4 or 5 digits in the square of two digit numbers ?

3. $300^2 = 90000$; $900^2 = 810000$

How many digits will there be in the square of three digit numbers ?

4. $2^2 = 4$; $12^2 = 144$; $30^2 = 900$; $300^2 = 90000$

If one digit is increased in a number then two digits will be increased in its square.

From this we know the following:

No. of digits in the number	No. of digits in its square root
1, 2	1
3, 4	2
5, 6	3

The given number is divided into periods of two digits from the unit place to find the number of digits in its square root.

$$\sqrt{6\mathring{7}4\mathring{8}5\mathring{9}}$$

There will be three digits in the square root of this number.

How many digits will be there in the square root of 2467596? 4.

Examples

1. Find the square root 6724.

$\mathring{6}7\mathring{2}4$ — There will be two digits in the square root

$80^2 = 6400$; $90^2 = 8100$

$\therefore$ The square root will be in between 80 and 90.

Let $80 < x < 90$

$6724 = (80 + x)^2 = 6400 + 2x.80 + x^2$

$2x.80 + x^2 = 6724 - 6400 = 324$

$x(2.80 + x) = 324$

$x(160 + x) = 324$

If $x = 2$

$2(160 + 2) = 324$

$\therefore \sqrt{6724} = 80 + 2 = 82$

The method of calculation may be shown as follow:

	80 + 2	or		82
	6724			6724
80	6400		8	64
	324			324
160 + 2	324		162	324
	0			0

We can make use of this method for any number.

2. Find the square root of 64516.

	254	
	6 45 16	
2	4	2^2
	2 45	
45	2 25	5 × 45
	20 16	
504	20 16	4 × 504
	0	

$\therefore \sqrt{64516} = 254$

Exercise 9—3

Find the square root of the following :

(a) 361 (b) 2809 (c) 1849 (d) 2209

(e) 3844 (f) 4356 (g) 8464 (h) 37249

(i) 92416 (j) 208849

Answers

(a) 19 (b) 53 (c) 43 (d) 47 (e) 62 (f) 66 (g) 92
(h) 193 (i) 304 (j) 457

9—4. Square root of a decimal fraction

$(0{\cdot}1)^2 = {\cdot}1 \times {\cdot}1 = {\cdot}01$; $\sqrt{{\cdot}01} = {\cdot}1$

$(1{\cdot}1)^2 = 1{\cdot}1 \times 1{\cdot}1 = 1{\cdot}21$; $\sqrt{1{\cdot}21} = 1{\cdot}1$

$(0{\cdot}22)^2 = {\cdot}22 \times {\cdot}22 = {\cdot}0484$; $\sqrt{0{\cdot}0484} = {\cdot}22$

From this we may conclude that in the square of a decimal fraction there will be twice the number of digits of the given decimal fraction.

Therefore if a decimal fraction is a perfect square, then there will be half the number of digits in its square root.

Note : The number of digits will always be even in the perfect square of a decimal fraction. Why ?

Examples

1. Find the square root of 20·8849.

	4·57
	20. 88 49
4	16
	4· 88
8·5	4· 25
	6349
90·7	6349
	0

Put the dot in the unit place. Divide the integral part from right to left and the fractional part from left to right into periods of two digits. Then find the square root as before.

$\sqrt{20 \cdot 8849} = 4 \cdot 57$

There will be two digits in the fractional part of the square root since the given number has 4 digits in the fractional part.

2. Find $\sqrt{0\ 0484}$

	22
	484
2	4
	84
42	84
	0

$\sqrt{484} = 22$

$\sqrt{\cdot 0484} = \cdot 22$

3. Find $\sqrt{0\ 001089}$

	33
	1089
3	9
	189
63	189
	0

$\sqrt{1089} = 33$

$\sqrt{0 \cdot 001089} = \cdot 033$

There are 6 decimal digits in the given number. There will be 3 digits in the square root. For two zeros in the square, we will have one zero in the square root.

Exercise 9—4

1. Find the square root of the following :

(a) 7·29 (b) 22·09 (c) 32·49 (d) 70·56 (e) 0·0289

(f) 281·9041 (g) 0·095481 (h) 0·009604 (i) 0·000784

(j) 6517·3329

2. The square root of 4160·25 is 64·5. Using this find the square root of the following:

(a) 41·6025 (b) 416025 (c) 0·416025 (d) 0·00416025

3. If $\sqrt{2} \doteq 1{\cdot}414$; $\sqrt{3} \doteq 1{\cdot}732$ and $\sqrt{5} \doteq 2{\cdot}236$ find the square root of the following numbers:

(a) 32 (b) 147 (c) 180 (d) 72 (e) 243 (f) 128

Answers

1. (a) 2·7 (b) 4·7 (c) 5·7 (d) 8·4 (e) 0·17 (f) 16·79 (g) 0·309 (h) 0·098 (i) 0·028 (j) 80·73.

2. (a) 6·45 (b) 645 (c) ·645 (d) ·0645

3. (a) 5·655 (b) 12·124 (c) 13·416 (d) 8·484 (e) 15·588 (f) 11·312.

9—5. Making use of the table

The table given on the next page will help us to find the square of a given number ranging from 1 to 99 and to find the square root of a number less than 1000. For other numbers we can find the square roots approximately.

A↓ \ B→	0	1	2	3	4	5	6	7	8	9
0	0	1	4	9	16	25	36	49	64	81
1	100	121	144	169	196	225	256	289	324	361
2	400	441	484	529	576	625	676	729	784	841
3	900	961	1024	1089	1156	1225	1296	1369	1444	1521
4	1600	1681	1764	1849	1936	2025	2116	2209	2304	2401
5	2500	2601	2704	2809	2916	3025	3316	3249	3364	3481
6	3600	3721	3844	3969	4096	4225	4356	4489	4624	4761
7	4900	5041	5184	5329	5476	5625	5776	5929	6084	6241
8	6400	6561	6724	6889	7056	7225	7396	7569	7744	7921
9	8100	8281	8464	8649	8836	9025	9216	9409	9604	9801

To find the square of a number:

Consider the numbers in A including 0 and 9 as tens and the nnmbers in B including 0 and 9 as units

1. Find the square of 38.

The meeting place of 3 in A and 8 in B gives the square of 38. i.e. 1444.

$\therefore \; 38^2 = 1444.$

2. Find $\sqrt{1849}$

The number of the row in which 1849 is placed is 4 (in A). The number of the column in which 1849 is placed is 3 (in B).

$\therefore \; \sqrt{1849} = 43.$

Exercise 9–5

1. FInd the square root of the following numbers with the help of the table:

(a) 576 (b) 3136 (c) 4624 (d) 7921 (e) 9801 (f) 8836.

2. Find the square of the following numbers using the table:

(a) 27 (b) 39 (c) 45 (d) 78 (e) 89.

3. Find the difference between two consecutive numbers in the column B. What is the speciality in this difference ?

Similarly find the difference between two consecutive numbers in the row A. What property do you notice ?

Say why it is so.

10. Terms — Series

You have studied how to find the sum of the first 'n' natural numbers.

$$1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

Exercise 10

Find the sum of the following series:

1. $1 + 2 + 3 + \dots + 15$
2. $1 + 2 + 3 + \dots + 50$
3. $12 + 13 + 14 + \dots + 32$
4. $43 + 44 + 45 + \dots + 76$
5. $4 + 8 + 12 + \dots + 44$
6. $5 + 10 + 15 + \dots + 65$

Answers

1. 120   2. 1275   3. 452   4. 2023   5. 264   6. 455

10—1. Sum of the first 'n' odd natural numbers

$$1 = 1 = 1^2$$

$$1 + 3 = 4 = 2^2$$

$$1 + 3 + 5 = 9 = 3^2$$

$$1 + 3 + 5 + 7 = 16 = ?$$

$$1 + 3 + 5 + 7 + 9 = ? = ?$$

How many odd numbers are there in each row? Is there any relation between the number of digits and the sum?

The square of the number of digits of each row is the sum of that row.

$$1 + 3 + 5 = 3^2 \qquad \left[\frac{1+5}{2} = 3\right]$$

$$1 + 3 + 5 + 7 + 9 = 5^2 \qquad \left[\frac{1+9}{2} = 5\right]$$

$$1 + 3 + 5 + \dots + 33 = 17^2 \qquad \left[\frac{1+33}{2} = 17\right]$$

Exercise 10—1

Find the sum of the following :

1. $1 + 3 + 5 + ... + 23$
2. $5 + 7 + 9 + ... + 35$
3. $9 + 11 + 13 + ... + 37$
4. $1 + 3 + 5 + ... + 47$
5. $7 + 9 + 11 + \quad + 33$

Answers

1. 144 2. 320 3. 345 4. 576 5. 280.

10—2. Sequence

Find the difference between the consecutive numbers in the sequence 1, 4, 7, 10, 13, ... Note that the difference between any two consecutive numbers is the same.

Look at the following sequences:

15, 11, 7, 3, ...

2, 6, 10, 14, ...

7, $5\frac{1}{2}$, 4, $2\frac{1}{2}$, ...

In each row the difference between two consecutive numbers namely — 4, 4, — $1\frac{1}{2}$ respectively remains the same.

If the difference between any two consecutive numbers in a row is constant then that row is called a sequence.

If 'a' is the first term of the sequence and the common difference between any consecutive numbers is 'd', then the sequence is given by

$a, \ a + d, \ a + 2d, \ ...$

To find the nth term of a given sequence:

Let us find the 8th term of the sequence

4, 7, 10, 13, ...

4, 7, 10, 13, 16, 19, 22, 25.

25 is the 8th term of the sequence.

To find the 25th term we can find it by writing the sequence continuously. Is there any alternative method?

Common difference is 3.

If we add the common difference to a term we will get the next term.

The first term $a = 4$

Common difference $d = 3$

The second term is $7 = 4 + 3 \times 1 = 4 + 3(2-1)$

The third term is $10 = 4 + (3 \times 2) = 4 + 3(3-1)$

The 4th term is $13 = 4 + (3 \times 3) = 4 + 3(4-1)$

What do you see in this?

To find the third term add twice the common difference to the first term.

To find the 4th term, add three (4—1) times the common difference to the first term.

Similarly the 8th term $= 4 + 3(8-1) = 4 + 21 = 25$

What is the 25th term?

$$4 + 3(25-1) = 4 + 3 \times 24 = 76$$

To find the n th term of a sequence,

$$t_n = a + (n - 1)d.$$

Examples

1. Find the 12th term in the sequence 5, 9, 13, 17, ...

The first term $a = 5$

Common difference $d = 4$

$n = 12$

12th term t_{12} $= 5 + (12-1)4$

$= 5 + 11 \times 4$

$= 49$

2. Find the 18th term in the sequence, 61, 58, 55, ...

$a = 61$

$d = 58-61 = -3$

$n = 18$

$\therefore$ 18th term $= 61 + 17(-3)$

$= 61-51$

$= 10$

Exercise 10—2

1. Find the 11th term in the sequence 4, 7, 10, ...
2. Find the 15th term in the sequence 6, 11, 16, ...
3. Find the 12th term in the sequence 74, 70, 66, ...
4. Find the 20th term in the sequence 69, 64, 59, ...
5. Find the 9th term in the sequence 6, $7\frac{1}{2}$, 9, ...
6. Find the 15th term in the sequence 34, 28, 22, ...
7. Find the 11th term in the sequence $\frac{7}{12}$, $\frac{2}{3}$, $\frac{3}{4}$, ...
8. Find the 10th term in the row 3, 5, 9, 17, ...
9. What is the general rule to find the nth term in question 8.

Answers

1. 34 2. 76 3. 30 4. -26 5. 18 6. -50
7. $1\frac{5}{12}$ 8. $2^{10} + 1$ 9. $2^n + 1$.

10—3. Sum of a sequence

(1) To find the sum of the sequence 4 + 7 + 10 + 13.

The first term is 4

The last term is 13

Common difference is 3.

$$\begin{array}{rl} S = & 4 + 7 + 10 + 13 \\ S = & 13 + 10 + 7 + 4 \\ \hline 2S = & 17 + 17 + 17 + 17 \end{array}$$

$$2S = 17 \times 4$$

$$\therefore \quad S = \frac{17 \times 4}{2} = 34 = \frac{4}{2}(4 + 13)$$

The number of terms in the sequence $\frac{13 - 4}{3} + 1 = 4$

(2) Find the sum of the sequence

$$2 + 4 + 6 + 8 + 10 + 12$$

$$\begin{array}{rl} S = & 2 + 4 + 6 + 8 + 10 + 12 \\ S = & 12 + 10 + 8 + 6 + 4 + 2 \\ \hline 2S = & 14 + 14 + 14 + 14 + 14 + 14 \end{array}$$

$$2S = 14 \times 6$$

$$S = \frac{14 \times 6}{2} = 42 = \frac{6}{2}(2 + 12)$$

The number of terms in the sequence

$$= \frac{12 - 2}{2} + 1 = 6$$

$\therefore$ Sum of the sequence

$$= \frac{\text{number of terms}}{2} \times (\text{1st term} + \text{last term})$$

First term is a ; common difference is d ; nth term is $a + (n - 1)d$.

First term + last term $= a + [a + (n - 1)d]$

$$\text{Sum} = \frac{n}{2}[2a + (n-1)d]$$

If 'd' is the common difference of the sequence $a_1, a_2, \ldots a_n$ then the number of terms $n = \frac{a_n - a_1}{d} + 1$

Exercise 10—3

Find the sum of the following sequences:

1. 2 + 5 + 8 + 11 + ... + 26
2. 4 + $5\frac{1}{2}$ + 7 + ... + 16
3. 3 + 7 + 11 + ... + 39
4. 48 + 43 + 38 + ... + 13
5. (—1) + (—3) + (—5) + ... + (—19)

6. The fifth term of a sequence is 18 and the sum of the first 10 terms is 230. Find the sequence.

Answers

1. 126 2. 90 3. 210 4. 244

5. — 100 6 — 22, —12, — 2, 8 ....

MATHEMATICS CLUB—ACTIVITY II

Let us try to find out the day with the help of the formula, if the date is given.

Formula :

$$A + (2{\cdot}6\,B - {\cdot}2) + \left(\frac{C}{4} + C\right) + \left(\frac{D}{4} - 2D\right)$$

A: Date.

B: The serial number of the month, with the first month as March. (April — 2, May — 3, ... January — 11, February — 12).

C: The last 2 digits of the year. If it is January or February, take the previous year. In 1967, C is 67; if the date is 19-2-1931, C is 30.

D: The first 2 digits in the year. For 1967, D is 19.

At every stage, take the whole number part only, if there is any fraction.

Divide the number obtained by 7, and find the remainder.

If the remainder is 0, the day is Sunday. If it is 1, it is Monday and so on.

Let us find out the day on which we became Independent.

The date is 15-8-1947.

A = 15; B = 6; C = 47; D = 19

$2{\cdot}6\ B - {\cdot}2 = 15{\cdot}6 - {\cdot}2 = 15{\cdot}4 = 15$

$\frac{C}{4} + C = \frac{47}{4} + 47 = 11 + 47 = 58$

$\frac{D}{4} - 2D = \frac{19}{4} - 38 = 4 - 38 = -34$

Sum = 15 + 15 + 58 − 34 = 54

Divide 54 by 7. The remainder is 5.

∴ The day is Friday.

3. ALGEBRA

1—1. Indices — Revision

Let us recall what we have studied about 'powers' and indices in the VIII Standard.

$2 \times 2 \times 2 \times 2 \times 2 = 2^5$

2 is the base. 5 is the index.

2^5 means: "2 multiplied by itself 5 times"

In 5^2, 5 is the base and 2 is the index.

5 has been multiplied by itself two times.

$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$

$5^2 = 5 \times 5 = 25$

In x^3, x is the base and 3 is the index.

x has been multiplied by itself 3 times.

$2^4 \times 2^3 = (2 \times 2 \times 2 \times 2) \times (2 \times 2 \times 2) = 2^7 = 2^{4+3}$

$3^5 \times 3^4 = (3 \times 3 \times 3 \times 3 \times 3) \times (3 \times 3 \times 3 \times 3)$
$= 3^9 = 3^{5+4}$

$a^3 \times a^2 = (a \times a \times a) \times (a \times a) = a^5 = a^{3+2}$

From this we can see that

$a^6 \times a^8 = a^{6+8} = a^{14}$

$x^5 \times x^{12} = x^{5+12} = x^{17}$

Can you tell what is $a^m \times a^n$?

$a^m \times a^n = a^{m+n}$

$2^5 \times 3^2 = (2 \times 2 \times 2 \times 2 \times 2) \times (3 \times 3) = 2^5 \times 3^2$

The base of 2^5 is 2 and that of 3^2 is 3.

Since the bases differ, the index law does not hold good. Hence note that the index law will hold good only when the bases are the same.

$$a^m \times a^n = a^{m+n}$$

Exercise 1—1

1. Write the following in the index form:

(a) $4 \times 4 \times 4$ (b) $7 \times 7 \times 7 \times 7 \times 7 \times 7$

(c) $a \times a \times a \times a$ (d) $x \times x \times x \times$ ... to 10 factors

(e) $p \times p \times p \times$ ...... to 'a' factors.

2. Expand:

(a) 6^5 (b) 3^3 (c) a^6 (d) x^{20} (e) p^x

3. Express the product in the index form:

(a) $5^6 \times 5^4$ (b) $x^{20} \times x^7$ (c) $p^{12} \times p^9$

(d) $3^4 \times 3^2 \times 3^3$ (e) $x^7 \times x^3 \times x^5$

(f) $a^x \times a^y$ (g) $a^m \times a^n \times a^p$

(h) $3^2 \times 3^2 \times 3^2 \times 3^2$ (i) $a^p \times a^q \times a^r \times a^s$

(j) $p^a \times p^b \times p^c \times p^d \times p^e$

4. Express the product in the index form:

(a) $2^5 \times 3^4$ (b) $7^6 \times 5^7$ (c) $2^3 \times 5^7 \times 2^8$

(d) $a^3 \times b^4 \times a^2 \times b^5$

(e) $a^2 \times b^4 \times a^3 \times b^5 \times c^2 \times c^4$

Answers

3. (a) 5^{10} (b) x^{27} (c) p^{21} (d) 3^9 (e) x^{15}
(f) a^{x+y} (g) a^{m+n+p} (h) 3^{10} (i) $a^{p+q+r+s}$ (j) $p^{a+b+c+d+e}$

4. (a) $2^5 \times 3^4$ (b) $7^6 \times 5^7$ (c) $2^5 \times 5^7$
(d) $a^5 \times b^7$ (e) $a^5 b^9 c^6$

1—2. Indices — Division — Revision

Let us learn about division of 'powers' in this lesson.

$$2^5 \div 2^2 = \frac{2 \times 2 \times 2 \times 2 \times 2}{2 \times 2} = 2 \times 2 \times 2$$

$$= 2^3 = 2^{5-2}$$

$$3^7 \div 3^5 = \frac{3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3}{3 \times 3 \times 3 \times 3 \times 3} = 3 \times 3$$

$$= 3^2 = 3^{7-5}$$

$$a^6 \div a^3 = \frac{a \times a \times a \times a \times a \times a}{a \times a \times a} = a \times a \times a$$

$$= a^3 = a^{6-3}$$

From this we find that

1. The dividend and the divisor have the same base.

2. The index of the dividend is greater than that of the divisor.

3. The difference between their indices is the index of the quotient.

Hence if $m > n$ can't we deduce that $a^m \div a^n = a^{m-n}$?

See whether the law holds good when $a = 0$

$$2^5 \div 3^2 = \frac{2 \times 2 \times 2 \times 2 \times 2}{3 \times 3} = 2^5 \div 3^2$$

The index law will not hold good when we divide numbers having different bases.

Let us see what the quotient will be when the divisor has a greater index than the dividend.

$$3^4 \div 3^6 = \frac{3 \times 3 \times 3 \times 3}{3 \times 3 \times 3 \times 3 \times 3 \times 3} = \frac{1}{3^2} = \frac{1}{3^{6-4}}$$

$$a^3 \div a^7 = \frac{a \times a \times a}{a \times a \times a \times a \times a \times a \times a} = \frac{1}{a^4} = \frac{1}{a^{7-3}}$$

Hence if $m < n$, $a^m \div a^n = \frac{1}{a^{n-m}}$ $(a \neq 0)$

$$a^m \div a^n = a^{m-n} \; (m > n)$$

$$= \frac{1}{a^{n-m}} \; (m < n)$$

$$a \neq 0$$

Exercise 1—2

1. **Simplify:**

(1) $7^{10} \div 7^5$ (2) $a^9 \div a^3$ (3) $x^{12} \div x^7$

(4) $a^p \div a^q \; (p > q)$ (5) $a^{3p} \div a^{2q} \; (p > q)$

(6) $7^5 \div 7^{10}$ (7) $a^3 \div a^9$ (8) $x^7 \div x^{12}$

(9) $a^p \div a^q \; (q > p)$ (10) $a^{3m} \div a^{5n} \; (n < m)$

2. When $m = n$, give different values to 'm' and find the valne of $a^m \div a^n$ for different bases. What can we infer when the index of 'a' is zero ?

Answers

1. (1) 7^5 (2) a^6 (3) x^5 (4) a^{p-q} (5) a^{3p-2q}

(6) $\frac{1}{7^5}$ (7) $\frac{1}{a^6}$ (8) $\frac{1}{x^5}$ (9) $\frac{1}{a^{q-p}}$ (10) $\frac{1}{a^{5n-3m}}$

2. 1

1—3. Indices — zero, negative indices

$a^3 \div a^3 = \frac{\not{a} \times \not{a} \times \not{a}}{\not{a} \times \not{a} \times \not{a}} = 1$; $a^3 \div a^3 = a^{3-3} = a^0 \; \therefore a^0 = 1$

$5^4 \div 5^4 = \frac{\not{5} \times \not{5} \times \not{5} \times \not{5}}{\not{5} \times \not{5} \times \not{5} \times \not{5}} = 1$; $5^4 \div 5^4 = 5^{4-4} = 5^0$

$\therefore 5^0 = 1$

$$a^m \div a^m = \frac{a \times a \times \text{......... to m factors}}{a \times a \times \text{......... to m factors}} = 1$$

$$a^m \div a^m = a^{m-m} = a^0; \quad \therefore a^0 = 1$$

From this we can learn that the 0^{th} power of any base except 0 is 1.

$$a^0 = 1$$
$$(a \neq 0)$$

We know that

$3 - 7 = -4; \quad 5 - 2 = 5 + (-2)$

$$a^3 \div a^7 = \frac{1}{a^4}\,;\; a^3 \div a^7 = a^{3-7} = a^{-4} \quad \therefore \frac{1}{a^4} = a^{-4}$$

$$2^5 \div 2^2 = 2^{5-2} = 2^{5+(-2)} = 2^5 \times 2^{-2} \quad \therefore \frac{1}{2^2} = 2^{-2}$$

Further

$$1 \div a^3 = a^0 \div a^3 = a^{0-3} = a^{-3}\,;\; \frac{1}{a^3} = a^{-3}$$

$$1 \div a^n = a^0 \div a^n = a^{0-n} = a^{-n}\,;\; \frac{1}{a^n} = a^{-n}$$

$$a^{-n} = \frac{1}{a^n}$$
$$(a \neq 0)$$

$$(5^3)^4 = 5^3 \times 5^3 \times 5^3 \times 5^3 = 5^{3+3+3+3} = 5^{12} = 5^{3\times4}$$

$$(3^5)^6 = 3^5 \times 3^5 \times 3^5 \times 3^5 \times 3^5 \times 3^5 = 3^{30} = 3^{5\times6}$$

$$(a^4)^2 = a^4 \times a^4 = a^8 = a^{4\times2}$$

$$(a^m)^3 = a^m \times a^m \times a^m = a^{3m} = a^{m\times3}$$

$$(a^m)^n = a^m \times a^m \times \ldots\ldots \text{ to n factors}$$
$$= a^{m+m+\ldots\ldots \text{ n times}} = a^{mn}$$

$$(a^m)^n = a^{mn}$$

Let us now consider about the powers of products.

$6 = 2 \times 3$

$6^4 = 6 \times 6 \times 6 \times 6$

$= 2 \times 3 \times 2 \times 3 \times 2 \times 3 \times 2 \times 3$

$= (2 \times 2 \times 2 \times 2) \times (3 \times 3 \times 3 \times 3)$

$= 2^4 \times 3^4$

i. e. $(2 \times 3)^4 = 2^4 \times 3^4$

Similarly we can find that

$(ab)^3 = ab \times ab \times ab = (a \times a \times a) \times (b \times b \times b)$

$= a^3 b^3$

Generally $(ab)^m = a^m b^m$

$$(ab)^m = a^m b^m$$

Exercise 1—3

1. Expand :

(a) $(2^5)^2$ (b) $(7^2)^3$ (c) $(a^3)^4$ (d) $(x^2)^m$

(e) $(p^a)^b$ (f) $(xy)^4$ (g) $(pq)^6$ (h) $(2a)^4$

(i) $(3p)^3$ (j) $(abc)^6$

2. Write as positive powers:

(a) 2^{-3} (b) 3^{-4} (c) 6^{-5} (d) a^{-5} (e) p^{-6}

(f) x^{-m} (g) a^{-7}

3. Expand $(a^3)^2$ and a^{3^2}. See whether they are equal. From this find out whether $(a^m)^n$ and a^{m^n} have the same index.

4. If m and n are positive integers other than 0 and 1, find the values of m and n for which $(a^m)^n$ will be equal to a^{1}

5. **Expand and rewrite as positive powers.**

(a) $(a^3b^2)^4$ (b) $(2a^2)^3$ (c) $(4x^3)^5$ (d) $(a^2b^4)^5$

(e) $(p^6q^5)^4$ (f) $(2a)^{-5}$ (g) $(ab)^{-m}$ (h) $(a^2b^3)^{-m}$

Answers

1. 2^{10}, 7^6, a^{12}, x^{2m}, p^{ab}. x^4y^4, p^5q^5, $16a^4$, $27p^3$, $a^6b^6c^6$.

2. $\frac{1}{2^5}$, $\frac{1}{3^4}$, $\frac{1}{6^5}$, $\frac{1}{a^3}$, $\frac{1}{p^6}$, $\frac{1}{x^m}$, $\frac{1}{a^7}$

3. a^6, a^0 4. 2

5. $a^{12} b^8$, $8a^b$, $1024x^{15}$, $a^8 b^{12}$, $p^{24}q^{20}$,

$\frac{1}{32a^5}$, $\frac{1}{a^mb^m}$, $\frac{1}{a^6b^6}$

2—1. Polynomials — Kinds of polynomials

Look at the following expressions.

1. $5x$
2. $-4x^2$
3. $3x^4-1$
4. $2x^2 + 4x^3 + 1$

5 is the coefficient of x in 5 x.

— 4 is the coefficient of x^2 in $-4x^2$

The first two are called monomials as each of them has a single term. The third one is a binomial and the fourth is a trinomial. The union of many monomials results in a polynomial. Polynomials are denoted by P(x), Q(x), R(x), etc.

Monomials, binomials and trinomials are some special cases of polynomials.

The expression is defined for x on the set from which the variable x takes values.

For example $2m^3 - 4m^3 + 3$ is defined for 'm' on the set of integers provided the variable 'm' takes values from the set of integers. We can see that the coefficients of all the terms of the above expression are integers.

If we take rational values then the trinomial $3y^2 - 2y - 5$ is said to be defined for y on the set of rational numbers. Though the coefficients of these expressions are integers we can take them as rational numbers as the set of integers is a subset of the set of rational numbers.

You known that $a^0 = 1$.

Since 5 can be considered as $5x^0$, we can take $5x^0$ as 5 provided $x \neq 0$.

5 is a constant.

$3x^2 - 4x + 5$ is a polynomial in x.

The highest degree of x is 2.

Hence the degree of the polynomial is 2.

In the same way the degree of $8 - 34x + 2x^4$ is 4.

The degree of $x + 3$ is 1.

Can you find the degree of the polynomial 5 ?

Note the following :

POLYNOMIAL	NAME	DEGREE
0	Zero	No degree
4	Constant function	0
$2x - 5$	Linear function	1
$3x^2 + 2x - 7$	Quadratic function	2
$2x^3 + 7$	Cubic function	3
$3x^5 - 4x^2 + 2x - 5$	A function in polynomial	5
$a_0 + a_1x + \dots + a_nx^n$	A function in polynomial	n

Exercise 2—1

1. What kind of polynomials are the following ?

   (a) $2x-7$ (b) p^2+3p+8 (c) $5a^2$ (d) $8x^3+2x-9$

   (e) 9 (f) $5-7x$ (g) -3 (h) $7x^2+2$

2. 5, $-2x$, a, $3x^3$, 2a, $-3p^4$, x^2, xp

Choose the needed one from the above and find the following:

(a) A binomial in 'x'.

(b) A trinomial in 'a'.

(c) A binomial in 'p'.

(d) A polynomial in x having the largest number of terms.

3. Find the degree of the following polynomials.

   (a) $2x-3+7x^5-x^6$ (b) $3x^8+7x^2-2x+5$

   (c) $4x^2$ (d) $2x^4+x^3-6x^7+3x^2-9$

Answers

3. (a) 6 (b) 8 (c) 2 (d) 7

2—2. Polynomials having more than one variable

In the last lesson we studied about expressions having one variable. In this lesson we shall study about expressions having more than one variable.

1. $3x+4y+5$
2. $2x^2+4xy+y^2+3x-4y+7$
3. $x^3+3x^2y^2+4x^2+5xy+4x+5y+6$
4. $x^2+y^2+z^2+3xy+4x^2y+2xyz$

In the first three expressions x, y are the variables and in the fourth one x, y, z are the variables.

In a polynomial having a single variable, we have defind the degree of the polynomial to be the exponent of the highest power of the variable to be found among the terms of the polynomial.

The degree of monomial having more than one variable is the sum of the exponents of the powers of the variables.

For example:

The degree of the monomial $3x^2 y^4$ is $2 + 4 = 6$.

The degree of $-5x^3 y^2 z^4$ is $3 + 2 + 4 = 9$.

The degree of $6xy$ is $1 + 1 = 2$.

To find the degree of a polynomial having more than one variable:

1. Find the degree of each of its terms.
2. Determine the highest one among them.

The highest one is the degree of the polynomial.

Example :

In the polynomial $2x^2 - 3xy + 4y^3 - 2x^2y^2$ the degree of $2x^2$ is 2. The degree of $-3xy$ is 2. The degree of $4y^3$ is 3. The degree of $-2x^2y^2$ is 4. Hence the degree of the polynomial is 4.

If we form polynomials whose variables take up real number values then $x \times y = y \times x$ is a true statement. Hence xy and yx are like terms.

Though x^2y and xy^2 have 3 as their degree, they are not like terms.

$x^2y = x \times x \times y$; $xy^2 = x \times y \times y$

x^2y^3z, zy^3x^2, y^3x^2z are all like terms.

We can simplify like terms alone by adding.

Example :

Simplify :

$$3x^2 + 4xy - 5y^2 - 7x^2 + 4x + 6yx + 8y^2 - 5xy - 3x + 7y$$

$$= (3 - 7)x^2 + (4 + 6 - 5)\,xy + (-5 + 8)y^2 + (4 - 3)\,x + 7y$$

$$= -4x^2 + 5xy + 3y^2 + x + 7y$$

Exercise 2—2

1. Find the degree.

   (a) $3x^2y^3$ (b) $-5xy^5z^6$ (c) $4x^6\,y^2\,z^5$

   (d) $2x^2\,y^7z$ (e) $-3xyz.$

2. Which of the following are like terms ?

   (a) $x^3\,y^2,\;\; y^3\,z^2$

   (b) $-3x^2\,y^4,\;\; -5y^4\,x^2$

   (c) $5x^3\,y^2\,z^4,\;\; 4y^2\,x^4\,z^4$

   (d) $8x^4\,yz^2,\;\; 6z^2\,x^4\,y$

   (e) $2x\,y^3\,z^2,\;\; -3x\,z^2\,y^3$

3. Find the degree of the following expressions:

   (a) $4x^3 - 3x^2\,y^2 + 2y^3 - 4xy^4$

   (b) $7 - 3xy + 2y^2 - 3x^3y$

   (c) $3x^2 - 2xy + 4xyz - 2x^3\,y^2\,z$

   (d) $ax^3 + bx^2\,y + cxy^2 + dy^3$

4. Simplify :

   (a) $2x^2 + 5xy - 4y^2 + 3x^2 - 2y^2 + 5x - 3xy + 4y^2 - 2x + 7$

   (b) $3x^3 + 5x^2y - 5xy^2 + 8y^3 - 4xy^2 - 6y^3 + 2x^3 + 3x^2y$

(c) $x^2 + 2xy + y^2 - x^2 - 2xy + y^2 + x^4$
$- 2xy + 2y^2$

(d) $x^3 + x^2y - xy^2 + 8y^3 - 4x^2y - 3xy^2$
$- 7y^3 + 3x^3$

Answers

1. (a) 5 (b) 12 (c) 10 (d) 10 (e) 3
2. (b), (d), (e).
3. (a) 5 (b) 4 (c) 6 (d) 3
4. (a) $5x^2 + 2xy - 2y^2 + 3x + 7$
   (b) $5x^3 + 8x^2y - 9xy^2 + 2y^3$
   (c) $x^4 + 4y^2 - 2xy$
   (d) $4x^3 - 3x^2y - 4xy^2 + y^3$

3—1. Polynomials — Evaluation

Recall : $3 + 5 = 8$; $2 \times 3 = 6$; $(+2)(-2) = -6$
$(-2)(-3) = +6$; $-5 + 3 = -2$;
$-5 + (-3) = -8$.

When a = 3,

$$a + 5 = 3 + 5 = 8$$
$$2a = 2 \times 3 = 6$$
$$-2a = = -2 \times 3 = -6$$
$$-5 + a = -5 + 3 = -2$$
$$-5 - a = -5 - 3 = -8$$
$$a^2 = a \times a = 3 \times 3 = 9$$
$$4a^2 = 4 \times a \times a = 4 \times 3 \times 3 = 36$$
$$-6a^2 = -6 \times a \times a = -6 \times 3 \times 3 = 54$$
$$a^3 = a \times a \times a = 3 \times 3 \times 3 = 27$$
$$2a^2 + 3a - 8 = 2(3)^2 + 3(3) - 8$$
$$= 2 \times 9 + 3 \times 3 - 8$$
$$= 18 + 9 - 8 = 19$$

When $a = -5$,

$$a + 5 = -5 + 5 = 0$$

$$3a = 3(-5) = -15$$

$$-3a = -3(-5) = +15$$

$$a^2 = (-5)(-5) = +25$$

$$5a^2 = 5 \times (-5)(-5) = 5 \times 25 = 125$$

$$-2a^2 = -2(-5)(-5) = -50$$

$$a^3 = (-5)(-5)(-5) = -125$$

$$3a^2 - 2a + 5 = 3(-5)^2 - 2(-5) + 5$$

$$= 3 \times 25 - 2 \times (-5) + 5$$

$$= 75 + 10 + 5 = 90$$

Thus we ean evaluate polynomials defined on the set of numbers. Let us now evaluate the polynomial.

Example 1 :

Evaluate $6a^2 - 5a + 3$ defined on the set $\left\{ \cdot 5, \sqrt{2}, \frac{2}{3} \right\}$

When $a = \cdot 5$,

$$6a^2 - 5a + 3 = 6(\cdot 5)^2 - 5(\cdot 5) + 3$$

$$= 6 \times \cdot 25 - 5 \times \cdot 5 + 3$$

$$= 1\cdot 5 - 2\cdot 5 + 3 = 2$$

When $a = \sqrt{2}$,

$$6a^2 - 5a + 3 = 6(\sqrt{2})^2 - 5(\sqrt{2}) + 3$$

$$= 6 \times 2 - 5\sqrt{2} + 3 \quad [(\sqrt{2})^2 = 2]$$

$$= 15 - 5\sqrt{2}$$

When $a = \frac{2}{3}$,

$$6a^2 - 5a + 3 = 6\left(\frac{2}{3}\right)^2 - 5\left(\frac{2}{3}\right) + 3$$

$$= 6 \times \frac{4}{9} - 5 \times \frac{2}{3} + 3$$

$$= \frac{8}{3} - \frac{10}{3} + 3 = \frac{7}{3}$$

Example :

When $x = 2$ and $y = -1$,

$$3x^4 - 4x^3y + 2x^2y^2 - 3xy^3 + 4y^4 + 2x^2y - 3xy^2 + 4x + 5y + 7$$

$$= 3(2)^4 - 4(2)^3(-1) + 2(2)^2(-1)^2 - 3(2)(-1)^3 + 4(-1)^4 + 2(2)^2(-1) - 3(^2)(-1)^2 + 4(2) + 5(-1) + 7$$

$$= 3 \times 16 - 4(8)(-1) + 2(4)(1) - 3(2)(-1) + 4(1) + 2\ 4)(-1) - 3(2)(1) + 4(2) + 5(-1) + 7$$

$$= 48 + 32 + 8 + 6 + 4 - 8 - 6 + 8 - 5 + 7$$

$$= 94$$

Exercise 3—1.

1. Some polynomials are given below. Find their values by choosing the elements of the set given by their side:

(a) $3x^2 - 5x + 6$ $\{2, -2, 1, -1, 0\}$

(b) $2x^3 - 4x^2 + 3x - 7$ $\{0, -1, 3, -2\}$

(c) $x^4 + x^2 + 2$ $\left\{\sqrt{2}, \sqrt{5}, -\sqrt{3}, \sqrt{\frac{2}{3}}\right\}$

(d) $3x^2 - 7x + 8$ $\left\{\frac{1}{2}, \frac{1}{3}, \frac{3}{4}, \frac{5}{2}\right\}$

2. By substituting x 3 and $y = -2$, find the values of the following expressions.

(a) $5x^2 - 2xy + 3y^2 - 4x + 2y + 3$

(b) $2x^3 - 3x^2y + 4xy^2 + 5y^3 - 2x^2 + 3xy + 2y^2 + 4x$
$- 5y + 2$

(c) $3x^3 + 5xy^2 - 2x^2y + 5x^2 - 7y^2 + 4x - 2$

(d) $x^0 + y^0$

Answers

1. (a) 8, 28, 4, 14, 6

(b) -7, -16, 20, -45

(c) 8, 32, 14, $\frac{28}{9}$

(d) $\frac{21}{4}$, 6, $\frac{71}{16}$, $\frac{37}{4}$

2. (a) 56 (b) 112 (c) 204 (d) 19.

4—1. Polynomials – Addition

(a) You know that $3 + 5 = 8$; $5 - 3 = 2$; $3 - 5 = -2$;
$-3 - 5 = -8$

Similarly,

$3a + 5a = 8a$; $5a - 3a = 2a$; $3a - 5a = -2a$;
$-3a - 5a = -8a$

3, 5, -5, -3, 8, -8, 2, -2 are the coefficients of the terms.

Further $3a^2 + 5a^2 = 8a^2$; $5a^2 - 3a^2 = 2a^2$;
$3a^2 - 5a^2 = -2a^2$; $-3a^2 - 5a^2 = -8a^2$

For simplifying polynomials, like terms are to be added. Like terms are simplified by adding the coefficients of the

terms. That is, the coefficient of the resulting term is the sum of the coeffiicients of the like terms.

Example :

$3a^2 - 8 + 2a^2 - 7a + 9 - 4a^2 + 10a - 6 + 5a$

$= [3a^2 + 2a^2 - 4a^2] + [-7a + 10a + 5a]$

$+ [-8 + 9 - 6)$

$= [3 + 2 - 4]\, a^2 + [-7 + 10 + 5]\, a + [-8 + 9 - 6]$

$= [a^2 + 8a - 5]$

$a^2 + 8a \neq 9a; \quad a^2 + 8a \neq 9a^2.$

Verify the validity of th:se statements by giving various values to 'a'.

(b) Let us now add two or more polynomials:

1. $(2x^2 + 3x - 5) + (5x^2 - 4x + 7)$

$= (2x^2 + 5x^2) + (3x - 4x) + (-5 + 7)$

$= (2 + 5)x^2 + (3 - 4)\, x + (-5 + 7)$

$= 7x^2 - x + 2$

$$\begin{array}{r} 2x^2 + 3x - 5 \\ 5x^2 - 4x + 7 \\ \hline 7x^2 - x + 2 \\ \hline \end{array}$$

2. $(5x^3 - 6x^2 + 8x - 9) + (3x^4 - 7x^3 + 9x - 8)$

$= 3x^4 + (5x^3 - 7x^3) + (-6x^2) + (8x + 9x)$

$+ (-9 - 8)$

$= 3x^4 + (5 - 7)\, x^3 + (-6)\, x^2 + (8 + 9)\, x$

$+ (-9 - 8)$

$= 3x^4 - 2x^3 - 6x^2 + 17x - 17$

$$\begin{array}{r} 0x^4 + 5x^3 - 6x^2 + 8x - 9 \\ 3x^4 - 7x^3 + 0x^2 + 9x - 8 \\ \hline 3x^4 - 2x^3 - 6x^2 + 17x - 17 \\ \hline \end{array}$$

Note : In the first expression there is no term containing x^4. Hence it is noted as $0x^4$. In the second expression there is no term containing x^2. Hence its ccefficient is taken as 0 and the term is taken as $0x^2$. By doing so we can make the two expressions have equal number of terms.

3. Add : $4x - 8x^2 + 7x^4 - 5$; $2x^4 - 5x^3 + 9x$;
$3x^2 - 7x - 8x^3 + 9$

Before addirg them up they are arrang d in descending powers of x. Then they are arranged in columns of like terms. The columns are then added up.

$$7x^4 + 0x^3 - 8x^2 + 4x - 5$$

$$2x^4 - 5x^3 + 0x^2 + 9x + 0$$

$$\underline{0x^4 - 8x^3 + 3x^2 - 7x + 9}$$

$$(7 + 2)\,x^4 + (-5 - 8)\,x^3 + (-8 + 3)\,x^2 + (4 + 9 - 7)\,x + (-5 + 9)$$

$$= 9x^4 - 13x^3 - 5x^2 + 6x + 4$$

Exercise 4—1

1. (a) Write the following in descending powers of x.

(i) $2x^2 - 7x^3 + 3x - 8$

(ii) $8 - 2x^2 + 9x - 7x^4 + 2x^3$

(iii) $3x + 5x^6 - 7x^2 + 5$

(iv) $3x^2 + 2x - 7x^3 + 6$

(b) Write the expressions keeping the powers in the ascending order.

(i) $2x^3 - 4x^2 - 8 + 3x$

(ii) $4x^2 - 8x^3 + 3x + 8$

(iii) $5x^2 + 7 - x^3 + 8x$

(iv) $3 - 2x^3 + 5x^2 - 7x$

2. Keep the like terms together and simplify.

(a) $2a^2 + 3a + 5a^2 - 2a + 5 - 4a^2$

(b) $2x^3 - 5 + 4x^2 - 3x^3 + 5x - 3x^2 + x^3 - 7$

(c) $5x^3 - 7x^2 + 2x^3 - 5x^2 + 4x - 6x^3 + 2x^2 - 5x + 8$

(d) $3a^3 - 4a - 5a^2 - 2a^3 + 8 - 6a + 10a^2 + 4a$

3. Add the following expressions.

(a) $(5x^3 + 4x^2 - 6x + 8), (2x^3 - 5x^2 + 4x - 6)$

(b) $2x^3 - 6x^2 + 3x - 4), (5x^3 + 2x^2 - 5x + 8)$

(c) $(3x^2 + 6x - 4), (2x^2 + 8x - 9), (12 - 4x - 7x^2)$

(d) $(2x^3 - 3x^2 + 4x - 7), (-3x^3 + 4x^2 + 3x + 8), (5 + 4x + 2x^3)$

(e) $(2x^3 + 4x - 5), (3x^2 - 5x + 2), (4x^3 - 2x^2 + 3)$

4. If the coefficient of x^2 in the sum of $5x - 9x^2 + 2x^3 - 7$; $4x^2 - 3x^3 - 5 + 6x$ and $3 + ax^2 + 9x - 2x^3$ is I, find the value of 'a'.

5. If the sum of $ax^3 - 8x^2 + 9x - 3$, $4x^3 + bx^2 + 6x + 9$, $2x^3 + 3x^2 - cx + d$ is $4x^3 + 3x^2 - 4x + 8$, find the value of a, b, c, d.

Answers

2. (a) $3a^2 + a + 5$

(b) $x^2 + 5x - 12$

(c) $x^3 - 10x^2 - x + 8$

(d) $a^3 + 5a^2 - 6a + 8$

3. (a) $7x^3 - x^2 - 2x + 2$

(b) $7x^3 - 4x^2 - 2x + 4$

(c) $-2x^2 + 10x - 1$

(d) $x^3 + x^2 + 11x + 6$

(e) $6x^3 + x^2 - x$

4. 6 5. $-2, 8, 19, 2$

4—2. Polynomials — zero, negative polynomials

Recall $8 + 0 = 8$; $-5 + 0 = -5$; $a + 0 = a$.

(a) If the coefficients of the terms of a polynomial are zero then the polynomial is called a **zero polynomial**.

Example :

$$0x^3 + 0x^2 + 0x + 0$$

is a zero polynomial of degree 3. In polynomials the zero polynomial possesses all the properties which the number 0 possesses in the number system.

$(8x^2 + 3x + 5) + (0x^2 + 0x + 0) = 8x^2 + 3x + 5$

$P(x) + 0(x) = P(x)$

Hence the zero polynomial is the additive identity element.

(b) We know that

$-5 + 5 = 0$; $8 + (-8) = 0$; $9 + (-9) = 0$;

$a + (-a) = 0$.

In this -5 is the additive inverse of 5, -8 is the additive inverse of 8, -9 is the additive inverse of 9 and $-a$ is the additive inverse of 'a'.

Let us now find out the additive inverse of a polynomial termed as the negative of a polynomial.

Example :

Let $ax^2 + bx + c$ be the negative polynomial of $5x^2 + 8x - 9$.

Then $(5x^2 + 8x - 9) + (ax^2 + bx + c) = 0x^2 + 0x + 0$

$\therefore (5 + a)x^2 + (8 + b)x + (-9 + c) = 0x^2 + 0x + 0$

$\therefore\ 5 + a = 0;$	$a = -5$	$5x^2 +$	$8x - 9$
$8 + b = 0;$	$b = -8$	$-5x^2 +$	$(-8)x + 9$
$-9 + c = 0;$	$c = 9$	$0x^2 +$	$0x + 0$

Hence the additive inverse of

$$5x^2 + 8x - 9 \text{ is } -5x^2 - 8x + 9$$

In general, if $P(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$ then, its additive inverse is

$$-P(x) = -a_0 - a_1x - a_2x^2 - \dots - a_nx^n$$

The negative of a polynomial is obtained by changing the sign of every term of a polynomial

The additive inverse of a polynomial is called the negative of that polynomial.

Verify : $P(x) + (-P(x)) = 0(x)$

$-P(x) + P(x) = 0(x)$

Exercise 4—2

Find the negative polynomials:

1. $4x^2 - 5x + 7$
2. $3x^3 - 5x^2 + 6x - 9$
3. $4x^3 - 3x^2 - 6x - 7$
4. $2x^3 + 3x^2 + 4x + 5$
5. $8x^2 - 9x + 6$

Answers

1. $-4x^2 + 5x - 7$
2. $-3x^3 + 5x^2 - 6x + 9$
3. $-4x^3 + 3x^2 + 6x - 7$
4. $-2x^3 - 3x^2 - 4x - 5$
5. $-8x^2 + 9x - 6$

4—3. Polynomials — Subtraction

Recall : In the real number system $8 - 8 = 8 + (-8)$

That is, subtracting a number from another is the same as adding the additive inverse of the subtrahend to the minuend.

Example 1 :

$P(x) = 3x^4 - 5x^3 + 7x^2 - 9x - 6$

$Q(x) = x^4 + 4x^3 - 8x^2 + 5x - 9$

Find $P(x) - Q(x)$

$P(x) - Q(x) = P(x) + [-Q(x)]$

$-Q(x) = -x^4 - 4x^3 + 8x^2 - 5x + 9$

Hence $P(x) - Q(x) = P(x) + [-Q(x)]$

$= (3x^4 - 5x^3 + 7x^2 - 9x - 6) + (-x^4 - 4x^3 + 8x^2 - 5x + 9)$

$= (3 - 1)x^4 + (-5 - 4)x^3 + (7 + 8)x^2 + (-9 - 5)x + (-6 + 9)$

$= 2x^4 - 9x^3 + 15x^2 - 14x + 3$

$$\begin{array}{r} 3x^4 - 5x^3 + 7x^2 - 9x - 6 \\ -x^4 - 4x^3 + 8x^2 - 5x + 9 \\ \hline 2x^4 - 9x^3 + 15x^2 - 14x + 3 \\ \hline \end{array}$$

Example 2 :

Subtract:

$$P(x) = 5x^4 + 3x^2 - 7x + 9$$

$$Q(x) = 2x^3 - 6x - 11$$

$$P(x) = 5x^4 + 0x^3 + 3x^2 - 7x + 9$$

$$-Q(x) = 0x^4 - 2x^3 + 0x^2 + 6x + 11$$

$$P(x) + [-Q(x)] = 5x^4 - 2x^3 + 3x^2 - \ x + 20$$

Note for what terms we have put 0 as the coefficient.

Exercise 4—3

1. Subtract the second expression from the first.

   (a) $(4x^4 + 3x^3 + 7x^2 + 8x + 6)$; $(2x^4 + x^3 + 5x^2 + 3x + 5)$

   (b) $(5x^3 + 8x^2 + 9x + 5)$; $(3x^3 + 6x + 2)$

   (c) $(5x^4 + 6x^3 + 3x^2 + 5)$; $(2x^4 + 3x^3 + 3x^2 + 5)$

   (d) $(4x^3 - 5x^2 - 6x - 9)$; $(3x^3 + 2x^2 + 3x + 4)$

   (e) $(2x^4 + 5x + 6x^2 + 6)$; $(3x^4 - 5x^2 + 7x + 6)$

   (f) $(5x^4 - 4x^3 - 5x + 7)$; $(3x^4 - 5x^2 + 7x + 6)$

2. On subtracting $3x^4 - 7x^3 - 6x^2 + 9x - 5$ from $8x^4 - 5x^3 + ax^2 + 9$ the coefficient of x^2 is 8. Find the value of 'a'.

3. On subtracting $3x^4 - px^3 - 5x^2 + qx - 4$ from $ax^4 - 4x^3 + bx^2 + 2x + c$ the expression $2x^4 + 5x^3 + 3x^2 - 7x - 9$ is got. Find the value of a, b. c, p and q.

Answers

1. (a) $2x^4 + 2x^3 + 2x^2 + 5x + 1$

   (b) $2x^3 + 8x^2 + 3x + 3$

   (c) $3x^4 + 3x^3$

   (d) $x^3 - 7x^2 - 9x - 13$

(e) $-x^4 + 5x^3 + 11x^2 - 7x + 12$

(f) $2x^4 - 4x^3 + 5x^2 - 12x + 1$

2. 2   3. 5, -2, -13, 9, 9,

4—4. Polynomials — Addition and subtraction

Example 1 :

If $P(x) = 2x^3 - 7x^2 + 9x - 3$

$Q(x) = 4x^3 + 3x^2 - 7x + 6$

$R(x) = 9x^2 - 3x + 5$

Find (a) $P(x) + Q(x) + R(x)$

(b) $P(x) + Q(x) - R(x)$

(c) $P(x) - Q(x) + R(x)$

(d) $Q(x) + R(x) - P(x)$

(a)

$$\begin{array}{rl} P(x): & 2x^3 - 7x^2 + 9x - 3 \\ Q(x): & 4x^3 + 3x^2 - 7x + 6 \\ R(x): & 0x^3 + 9x^2 - 3x + 5 \\ \hline \therefore P(x) + Q(x) + R(x): & 6x^3 + 5x^2 - x + 8 \end{array}$$

(b)

$$\begin{array}{rl} P(x): & 2x^3 - 7x^2 + 9x - 3 \\ +Q(x): & 4x^3 + 3x^2 - 7x + 6 \\ -R(x): & 0x^3 - 9x^2 + 3x - 5 \\ \hline \therefore P(x) + Q(x) - R(x): & 6x^3 - 13x^2 + 5x - 2 \end{array}$$

(c)

$$\begin{array}{rl} P(x): & 2x^3 - 7x^2 + 9x - 3 \\ -Q(x): & -4x^3 - 3x^2 + 7x - 6 \\ R(x): & 0x^3 + 9x^2 - 3x + 5 \\ \hline P(x) - Q(x) + R(x): & -2x^2 - x^2 + 13x - 4 \end{array}$$

(d)

$$\begin{array}{rl} Q(x): & 4x^3 + 3x^2 - 7x + 6 \\ R(x): & 0x^3 + 9x^2 - 3x + 5 \\ -P(x): & -2x^3 + 7x^2 - 9x + 3 \\ \hline Q(x) + R(x) - P(x): & 2x^3 + 19x^2 - 19x + 14 \end{array}$$

Example 2 :

What should be added to $4x^3 - 7x^2 - 2x + 9$ to get $6x^3 - 3x^2 + 9x - 8$?

Let P (x) be $4x^3 - 7x^2 - 2x + 9$ and

R (x) : $6x^3 - 3x^2 + 9x - 8$

Let the polynomial to be added be Q (x)

Then $P(x) + Q(x) = R(x)$

$\therefore \quad Q(x) = R(x) - P(x)$

$$\begin{array}{rl}
R(x) : & 6x^3 - 3x^2 + 9x - 8 \\
-P(x) : & \underline{-4x^3 + 7x^2 + 2x - 9} \\
R(x) - P(x) : & \underline{2x^3 + 4x^2 + 11x - 17}
\end{array}$$

$\therefore$ The polynomial to be added is

$2x^3 + 4x^2 + 11x - 17$

Example 3:

By how much does

$7x^2 - 3x + 9$ exceed $5x^3 + 6x - 4$?

P (x) : $7x^2 - 3x + 9$; R (x) : $5x^3 + 6x - 4$

If the subtrahend is taken as Q (x),

$P(x) - Q(x) = R(x)$

$\therefore \quad -Q(x) = R(x) - P(x)$

or $\quad Q(x) = P(x) - R(x)$

$$\begin{array}{rl}
P(x) : & 0x^3 + 7x^2 - 3x + 9 \\
-R(x) : & \underline{-5x^3 + 0x^2 - 6x + 4} \\
P(x) - R(x) : & \underline{-5x^3 + 7x^2 - 9x + 13}
\end{array}$$

The subtrahend is $\underline{-5x^3 + 7x^2 - 9x + 13}$

Exercise 4—4

1. Simplify :

(a) $(2x^3 + 4x^2 + 5x + 6) + (x^3 + 3x^2 + 2x + 1) + (3x^3 - x^2 + x - 4)$

(b) $(x^3 - 4x^2 + 4x - 2) + (2x^3 - 3x^2 - 2x - 1)$
$+ (x^3 - x^2 - x + 1)$

(c) $x^4 - 3x^3 + 2x^2 - x + 1) + (x^3 - 2x^2 - 5x + 1)$
$- (2x^3 - x^2 + x - 1)$

(d) $(3x^3 - 5x^2 + 4x - 1) + (4x^3 - 2x^2 + 5x - 2)$
$- (5x^3 - 3x^2 - x - 2)$

(e) $(4x^3 - x^2 - x - 2) - (x^3 + 2x^2 - 2x - 3)$
$+ (x^3 - 2x^2 - 2x - 1)$

(f) $(5x^3 - 3x^2 + 2x - 1) - (x^3 - 2x^2 - 5x + 2)$
$+ (x^3 + x^2 + x + 1)$

(g) $(x^3 + 2x^2 - 2x - 1) - (x^3 - x^2 - x - 1)$
$- (2x^3 - 2x^2 + x - 1)$

(h) $(2x^3 - 3x^2 - x + 2) - (3x^3 - 2x^2 + x - 2)$
$- (2x^3 - 5x^2 + x + 5)$

(i) $(3x^3 - x - 2) - (x^2 - 5x + 4) + (4x^4 - 3x + 1)$

(j) $(5x^3 - 4x^2 + 4) - (3x^4 - x - 2)$
$- (x^3 - 3x^2 + 2x + 1)$

2. In the following problems, what should be subtracted from the first polynomial to get the second one?

(a) $(x^3 + 2x^2 - x - 4)$; $(2x^3 + 5x^2 - 2x - 1)$

(b) $(2x^3 - 3x^2 + x - 1)$; $(3x^3 - 4x^2 + 2x - 2)$

(c) $(3x^3 - 4x^2 - x - 1)$; $(2x^3 - x^2 + 5x - 2)$

(d) $(x^4 - x^2 - x + 1)$; $(x^3 + x^2 + x + 1)$

(e) $(2x^4 - 3x^2 + x - 1)$; $(2x^4 - x^2 - x + 1)$

3. What should be added to the first to get the second polynomial?

(a) $(x^3 + 2x^2 - x - 4)$, $(2x^3 + 5x^2 - 2x + 1)$

(b) $(2x^3 + 3x^2 - x - 1)$, $(3x^3 - 4x^2 - 2x + 2)$

(c) $(3x^3 - 4x^2 + x - 1)$, $(2x^3 + x^2 - 5x + 2)$

(d) $(x^4 - x^2 + x - 1)$, $(x^3 - x^2 - x + 1)$

(e) $(2x^4 + 3x^3 - x + 1)$, $(2x^4 - x^2 + x - 1)$

4. Find the coefficient of x in the following :

(a) $(x^2 + 2x + 1) + (x^3 - x^2 + 1)$
$+ (x^2 + x^3 + 2x + 1)$

(b) $(2x^3 - x^2 + 1) + (x^3 - 2x^2 + 1) + (x^2 + x + 1)$

(c) $(x^3 - 3x^2 - 2x + 1) - (x^3 - 2x^2 - x - 1)$
$- (2x^3 - x^2 - x + 1)$

(d) $(2x^3 + 3x^2 - x + 1) - (x^3 - x^2 - x + 1)$
$- (2x^3 - 2x^2 + x - 1)$

Answers

1. (a) $6x^3 + 6x^2 + 8x + 3$
   (b) $4x^3 - 8x^2 + x - 2$
   (c) $x^4 - 4x^3 + x^2 - 7x + 3$
   (d) $2x^3 - 4x^2 + 10x - 1$
   (e) $4x^3 - 5x^2 - x$
   (f) $5x^3 + 8x - 2$
   (g) $-2x^3 + 5x^2 - 2x - 1$
   (h) $-3x^3 + 4x^2 - 3x - 1$
   (i) $4x^4 + 3x^3 - x^2 + x - 5$
   (j) $-3x^4 + 4x^3 - x^2 + x + 5$

2. (a) $x^3 - 3x^2 + x - 3$
   (b) $-x^3 + x^2 - x + 1$
   (c) $x^3 - 3x^2 - 6x + 1$
   (d) $x^4 - x^3 - 2x^2 - 2x$
   (e) $-2x^2 + 2x - 2$

3. (a) $x^3 + 2x^2 - x + 5$
   (b) $x^3 - 7x^2 - x + 3$
   (c) $-x^3 + 5x^2 - 6x + 3$
   (d) $-x^4 + x^3 - 2x + 2$
   (e) $-4x^2 + 2x - 2$

4. (a) 4
   (b) 1
   (c) 0
   (d) -1

5—1. Polynomials — Multiplication by a number

If $P(x) = 3x^2 - 4x + 2$, find $P(x) + P(x)$; $P(x) + P(x) + P(x)$.

Is there any relationship between the coefficients of the terms of $P(x)$ and those of $P(x) + P(x)$; $P(x) + P(x) + P(x)$?

$$P(x) + P(x) = (3x^2 - 4x + 2) + (3x^2 - 4x + 2)$$
$$= 6x^2 - 8x + 4$$

$$P(x) + P(x) + P(x) = (3x^2 - 4x + 2) + (3x^2 - 4x + 2) + (3x^2 - 4x + 2)$$
$$= 9x^2 - 12x + 6$$

Hence we can find that $P(x) + P(x)$ is a polynomial got by multiplying $P(x)$ by 2 and $P(x) + P(x) + P(x)$ is got by multiplying $P(x)$ by 3.

Hence $2P(x) = P(x) + P(x)$

$3P(x) = P(x) + P(x) + P(x)$

In general, we can deduce that

$mP(x) = P(x) + P(x) + \ldots\ldots$ m times

$mP(x)$ is a polynomial got by multiplying each term of $P(x)$ by 'm' This property can be used to multiply a polynomial

by any real number. Hence the different terms of the product of a polynomial and a real number can be obtained by multiplying the corresponding like terms of the given polynomial by the given real number.

Example :

If P (x) is $3x^4 - 5x^2 + 7x - 9$ find 5 P (x), $\frac{4}{7}$ P (x), -3 P (x), $\sqrt{2}$ P (x).

$$5\,P(x) = 5 \times 3x^4 + 5 \times (-5x^2) + 5 \times 7x + 5 \times (-9)$$
$$= 15x^4 - 25x^2 + 35x - 45$$

$$\frac{4}{7}\,P(x) = \frac{4}{7} \times 3x^4 + \frac{4}{7} \times (-5x^2) + \frac{4}{7} \times 7x + \frac{4}{7} \times (-9)$$
$$= \frac{12}{7}x^4 - \frac{20}{7}x^2 + 4x - \frac{36}{7}$$

$$-3\,P(x) = -3 \times 3x^4 + (-3)(-5x^2) + (-3)(7x) + (-3)(-9)$$
$$= -9x^4 + 15x^2 - 21x + 27$$

$$\sqrt{2}\,P(x) = \sqrt{2} \times 3x^4 + \sqrt{2} \times (-5x^2) + \sqrt{2}\,(7x) + \sqrt{2}\,(-9)$$
$$= 3\sqrt{2}\,x^4 - 5\sqrt{2}\,x^2 + 7\sqrt{2}\,x - 9\sqrt{2}$$

Exercise 5—1

Multiply the following polynomials by the numbers given against them:

1. $2x^2 - 5x + 7$ $\left(3, -5, \sqrt{5}, \frac{1}{4}\right)$
2. $x^3 - 5x^2 + 3x - 2$ $\left(2, -7, -\frac{3}{4}, \sqrt{2}\right)$
3. $5 - 7x + x^2 - 2x^3$ $\left(-6, \frac{2}{5}, \frac{\sqrt{2}}{5}, -\frac{5}{2}\right)$
4. $4x^3 - 5x^2 - 7x + 9$ $\left(3, \frac{3}{7}, \frac{\sqrt{5}}{2}, -\frac{7}{2}\right)$
5. $\frac{2}{7}x^3 - \frac{3}{7}x^2 - \frac{4}{7}x + \frac{5}{7}$ $\left(7, \frac{7}{2}, -\frac{7}{3}, \frac{7}{5}\right)$

6. Multiply the above polynomials by 1 and find out the function of 1 in the multiplication of polynomials.

Answers

1. $6x^2 - 15x + 21$; $\quad -10x^2 + 25x - 35$;

   $2\sqrt{5}x^2 - 5\sqrt{5}\,x + 7\sqrt{5}$; $\frac{1}{2}x^2 - \frac{5}{4}x + \frac{7}{4}$

2. $2x^3 - 10x^2 + 6x - 4$;

   $-7x^3 + 35x^2 - 21x + 14$;

   $-\frac{3}{4}x^3 + \frac{15}{4}x^2 - \frac{9}{4}x + \frac{3}{2}$;

   $\sqrt{2}x^3 - 5\sqrt{2}x^2 + 3\sqrt{2}\,x - 2$

3. $-30 + 42x - 6x^2 + 12x^3$;

   $2 - \frac{14}{15}x + \frac{2}{5}x^2 - \frac{4}{5}x^3$;

   $\sqrt{2} - \frac{7\sqrt{2}}{5}x + \frac{\sqrt{2}}{5}x^2 - \frac{2\sqrt{}}{5}x^3$;

   $\frac{-25}{2} + \frac{35}{2}x - \frac{5}{2}x^2 - 5x^3$

4. $12z^3 - 15x^2 - 21x + 27$;

   $\frac{12}{7}x^3 - \frac{15}{7}x^2 - 3x^1 + \frac{27}{7}$;

   $2\sqrt{5}\,x^3 - \frac{5\sqrt{5}}{2}x^2 - \frac{7\sqrt{5}}{2}x + \frac{9\sqrt{5}}{2}$;

   $-14x^3 + \frac{35}{2}x^2 + \frac{49}{2}x - \frac{63}{2}$

5. $2x^3 - 3x^2 - 4x + 5$;

   $x^3 - \frac{3}{2}x^2 - 2x + \frac{5}{2}$;

   $-\frac{2}{3}x^3 + x^2 + \frac{4}{3}x - \frac{5}{3}$;

   $\frac{2}{5}x^3 - \frac{3}{5}x - \frac{4}{5}x + 1$

6. Multiplicative dentity

5—2. Polynomials — Multiplying by Monomials

Recall :

$x^2 \times x^3 = x^5$; $x \times x^3 = x^4$; $5 \times x^2 = 5x^2$.

In this lesson let us learn how to multiply a polynomial by a monomial.

Multiply $P(x)$: $3x^2 - 4x + 7$ by 2, x, x^2, x^3, $-x^2$, $3x^2$, $-7x^3$

$$2 \times P(x) = 2 \times (3x^2 - 4x + 7)$$
$$= 6x^2 - 8x + 14x$$

$$x \times P(x) = x \times (3x^2 - 4x + 7)$$
$$= x \times 3x^2 + x(-4x) + x(7)$$
$$= 3x^3 - 4x^2 + 7x$$

$$x^2 \times P(x) = x^2 \times (3x^2 - 4x + 7)$$
$$= x^2 \times 3x^2 + x^2(-4x) + x^2(7)$$
$$= 3x^4 - 4x^3 + 7x^2$$

$$x^3 \times P(x) = x^3 \times (3x^2 - 4x + 7)$$
$$= x^3 \times 3x^2 + x^3(-4x) + x^3(7)$$
$$= 3x^5 - 4x^4 + 7x^3$$

$$-x^2 \times P(x) = -x^2 \times (3x^2 - 4x + 7)$$
$$= (-x^2) \times 3x^2 + (-x^2)(-4x) + (-x^2)(7)$$
$$= -3x^4 + 4x^3 - 7x^2$$

$$3x^2 \times P(x) = 3x^2 \times (3x^2 - 4x + 7)$$
$$= 3x^2 \times (3x^2) + 3x^2(-4x) + 3x^2(7)$$
$$= 9x^4 - 12x^3 + 21x^2$$

$- 7x^3 \times P(x) = - 7x^3 \times (3x^2 - 4x + 7)$

$= - 7x^3 (3x^2) + (- 7x^3)(- 4x) + (- 7x^3)(7)$

$= - 21x^5 + 28x^4 - 49x^3$

Hence to multiply a polynomial by a monomial we have to multiply every term of the polynomial by the monomial.

Exercise 5—2 (Mental sums)

Multiply the following polynomials by the monomials given on the right-hand side.

1. $2x - 5$ $(3, x, x^3, x^4, 2x^2, - 5x^3)$

2. $4x^2 - 3x + 2$ $(- 3x, 4x^3, \frac{7}{2}x^2, \sqrt{2}x)$

3. $x^3 - 4x^2 - 5x + 6$ $(- 5x^2, 3x, \sqrt{5}x^2, \frac{7}{2}x)$

4. $2x^2 - 7x + 9$ $(- x, - 4x^3, 3x^2, 5x)$

5—3. Polynomials - Multiplication by Binomial

Recall:

$(3 + 5) 7 = 3 \times 7 + 5 \times 7$

$(x^2 - 3x + 5) \times 7 = 7x^2 + 7(- 3x) + 7(5)$

We know that the distributive property of multiplication over addition holds good when we multiply a polynomial by a monomial. Making use of the same distributive property, we can multiply a polynomial by another polynomial. Now we shall learn to multiply a polynomial by a binomial of first degree.

Example 1:

$(3x^2 - 4x - 7)(2x + 3)$

$= (3x^2 - 4x - 7) \times 2x + (3x^2 - 4x - 7) \times 3$

$= 2x(3x^2) + 2x(- 4x) + 2x(- 7) + 3(3x^2)$

$+ 3(- 4x) + 3(- 7)$

$= 6x^3 - 8x^2 - 14x + 9x^2 - 12x - 21$

$= 6x^3 + (-8 + 9)x^2 + (-14 - 12)x - 21$

$= 6x^3 + x^2 - 26x - 21$

$$\begin{array}{r} 3x^2 - 4x - 7 \\ 2x + 3 \\ \hline 6x^3 - 8x^2 - 14x \\ 9x^2 - 12x - 21 \\ \hline 6x^3 + x^2 - 26x - 21 \\ \hline \end{array}$$

Example 2 :

$(2x^3 - 5x - 6)\ (3 - 2x)$

$= (2x^3 - 5x - 6) \times 3 + (2x^3 - 5x - 6) \times (-2x)$

$= 3(2x^3) + 3(-5x) + 3(-6) + (-2x)(2x^3) + (-2x)(-5x) + (-2x)(-6)$

$= 6x^3 - 15x - 18 - 4x^4 + 10x^2 + 12x$

$= -4x^4 + 6x^3 + 10x^2 + (-15 + 12)x - 18$

$= -4x^4 + 6x^3 + 10x^2 - 3x - 18$

$$\begin{array}{r} 2x^3 + 0x^2 - 5x - 6 \\ - 2x + 3 \\ \hline -4x^4 + 0x^3 + 10x^2 + 12x \\ 6x^3 + 0x^2 - 15x - 18 \\ \hline -4x^4 + 6x^3 + 10x^2 - 3x - 18 \\ \hline \end{array}$$

Exercise 5—3

1. Multiply the polynomials given on the left-hand side by each one of the binomials given on the right-hand side.

(a) $4x^2 - 5x + 9$; $x + 5$, $x + 3$, $x - 7$, $x - 9$

(b) $2x^3 + 5x^2 - 7x - 4$; $2x + 3$, $3x + 2$, $2x - 3$, $3x - 2$

(c) $5x^3 + 2x - 6$; $3 - x$, $5 - x$, $3 - 2x$, $5 - 2x$

(d) $3x^3 - 5x^2 + 7$; $3x + 5$, $2x - 7$, $5 - 3x$, $9 - 5x$

(e) $4x^3 - 3x - 9$; $2x + 5$, $3x - 7$, $4 - 5x$, $3 + 2x$

2. In the products of the polynomials given below find the coefficient of x^2 and x without doing the multiplication in full.

(a) $(4x^3 - 5x^2 + 7x - 9)(3x + 7)$

(b) $(2x^3 - 5x + 6)(4x - 5)$

(c) $(3x^3 + 4x^2 - 9)(2x - 11)$

(d) $(5x^2 + 3x - 7)(5x + 7)$

(e) $(2x^2 - 4x - 7)(7x - 9)$

Answers

1. (a) $4x^3 + 15x^2 - 16x + 45$;
$4x^3 + 7x^2 - 6x + 27$;
$4x^3 - 33x^2 + 44x - 63$;
$4x^3 - 41x^2 + 54x - 81$.

(b) $4x^4 + 16x^3 + x^2 - 29x - 12$;
$6x^4 + 19x^3 - 11x^3 - 26x - 8$;
$4x^4 + 4x^3 - 29x^2 + 13x + 12$;
$6x^4 + 11x^3 - 31x^2 + 2x + 8$.

(c) $-5x^4 + 15x^3 - 2x^2 + 12x - 18$;
$-5x^4 + 25x^3 - 2x^2 + 16x - 30$;
$-10x^4 + 15x^3 - 4x^2 + 18x - 18$;
$-10x^4 + 25x^3 - 4x^2 + 22x - 30$.

(d) $9x^4 - 25x^2 + 21x + 35$;
$6x^4 - 31x^3 + 35x^2 + 14x - 49$;
$-9x^4 + 30x^3 - 25x^2 - 21x + 35$;
$-15x^4 + 42x^3 - 45x^2 - 35x + 63$.

(e) $8x^4 + 20x^3 - 6x^2 - 33x - 45$;
$12x^4 - 28x^3 - 9x^2 - 6x - 63$;
$-20x^4 + 16x^3 + 15x^2 + 33x - 36$;
$8x^4 + 12x^3 - 6x^2 - 27x - 27$.

2. (a) $-14, 22$ (b) $-20, 49$ (c) $-44, -18$
(d) $50, -14$ (e) $-46, -13$

5—4. Polynomials — Multiplication by another Polynomial

In the previous lessons, we learnt about the use of the distributive law in finding the product of the polynomials. Let us try to extend its use.

Example 1 :

$(3x^3 - 7x^2 + 6)(2x^2 - 3x - 5)$

$= (3x^3 - 7x^2 + 6)\,2x^2 + (3x^3 - 7x^2 + 6)(-3x) + (3x^3 - 7x^2 + 6)(-5)$

$= 6x^5 - 14x^4 + 12x^2 - 9x^4 + 21x^3 - 18x - 15x^3 + 35x^2 - 30$

$= 6x^5 + (-14 - 9)\,x^4 + (21 - 15)\,x^3 + (12 + 35)\,x^2 + (-18)\,x - 30$

$= 6x^5 - 23x^4 + 6x^3 + 47x^2 - 18x - 30$

$$\begin{array}{rrrrrr}
3x^3 - & 7x^2 + & 0x + & 6 & & \\
 & 2x^2 - & 3x - & 5 & & \\
\hline
6x^5 - 14x^4 + & 0x^3 + & 12x^2 & & & \\
- 9x^4 + & 21x^3 + & 0x^2 - & 18x & & \\
 & - 15x^3 + & 35x^2 + & 0x - & 30 & \\
\hline
6x^5 - 23x^4 + & 6x^3 + & 47x^2 - & 18x - & 30 &
\end{array}$$

Example 2 :

Find the coefficients of x^3, x^2 in the product of $(4x^3 - 7x^2 + 9x - 5)$ and $(3x^2 - 4x + 5)$ without doing the multiplication in full.

The combinations leading to the term containing x^3:

$$(4x^3 - 7x^2 + 9x - 5)(3x^2 - 4x + 5)$$

The term containing x^3 in the first × the independent term in the second $= 4x^3 \times 5 = 20x^3$

The term containing x^2 in the first $\times$ the term containing x in the second $= -7x^2 \times -4x = 28x^3$

The term containing x in the first $\times$ the term containing x^2 in the second $= 9x \times 3x^2 = 27x^3$

The independent term in the first $\times$ the term containing x^3 in the second $= -5 \times 0x^3 = 0x^3$

The coefficient of x^3 in the product
$= 20 + 28 + 27 + 0 = 75$

The combinations leading to the terms containing x^2:

$$(4x^3 - 7x^2 + 9x - 5)\ (3x^2 - 4x + 5)$$

The term containing x^2 in the first $\times$ the independent term in the second $= -7x^2 \times 5 = -35x^2$

The term containing x in the first $\times$ the term containing x in the second $= 9x \times -4x = -36x^2$

The independent term in the first $\times$ the term containing x^2 in the second $= -5 \times 3x^2 = -15x^2$

The coefficient of x^2 in the product
$= -35 - 36 - 15 = -86$

Exercise 5—4

1. Multiply the polynomials given on the LHS by each one of those given on the RHS.

(a) $2x^3 - 3x^2 + 4x - 7$; $x^2 - 2x + 3$, $2x^2 + 3x + 1$, $4x^2 - 5$, $2x^2 + 7$

(b) $5x^3 - 6x^2 + 2x + 3$; $2x^2 + x - 5$, $4x^2 - 7x - 8$, $2x^2 + 5x + 7$, $3x^2 + 7x$

(c) $3x^3 + 7x - 5$; $x^2 - 3x + 4$, $2x^3 + 5$, $3x^2 + 5x - 7$, $5x^2 - 3x + 7$

(d) $2x^3 + 4x^2 - 9$; $3x^2 + 4x + 5$, $2x^2 - 4x + 5$,
$2x^2 - 7$, $3 - 2x - x^2$

(e) $4x^3 - 5x^2 + 8$; $3 - 4x - 2x^2$, $2 + 5x - x^2$,
$1 - x - x^2$, $1 + x - x^2$

2. In the product of the following polynomials find the coefficients of x^3 and x^2.

(a) $(2x^3 + 4x^2 + 1)(8x - 6)$

(b) $(4x^4 - 3x^3 + 6x^2 + 2)(2x^3 - 5x^2 - 3x + 2)$

(c) $(5x^5 - 3x^4 - x^2)(2x + 4)$

3. In the product of $(3x^3 - 5x^2 + 6x - 7)$ and $(ax^2 + 2x + b)$, the coefficient of x^4 is -4 and the coefficient of x is 4. Find the coefficients of x^5 and x^3.

4. The independent term in the product of $(2x^3 + ax^2 + 4x + 8)$ and $(4x^2 - bx + c)$ is 24 and the coefficients of x and x^4 are 4 and 8 respectively. Find the values of a, b and c.

Answers

1. (a) $2x^5 - 7x^4 + 16x^3 - 24x^2 + 26x - 21$;
$4x^5 + x^3 - 5x^2 - 17x - 7$;
$8x^5 - 12x^4 + 6x^3 - 13x^2 - 20x + 35$;
$4x^5 - 6x^4 + 22x^3 - 35x^2 + 28x - 49$.

(b) $10x^5 - 7x^4 - 27x^3 + 38x^2 - 7x - 15$;
$20x^5 - 59x^4 + 10x^3 + 46x^2 - 37x - 24$;
$10x^5 + 13x^4 + 9x^3 - 26x^2 + 29x + 21$;
$15x^5 - 17x^4 + 36x^3 + 23x^2 + 21x$.

2. (a) 20, -24 (b) -20, 2 (c) -2, -4

3. 6, 11 4. $a = \frac{5}{2}$, $b = 1$, $c = 3$.

6—1. Polynomials — Division by a monomial

Remember : $x^5 \div x^2 = x^3;\quad x^6 \div x^4 = x^2$

Observe the following divisions:

1. $x^2 \div 2 = \dfrac{x^2}{2}$ or $\dfrac{1}{2}x^2$

2. $3x^2 \div x = \dfrac{3x^2}{x} = 3x$

3. $12x^6 \div 3x^4 = \dfrac{12x^6}{3x^4} = 4x^2$

4. $15x^2 y^2 \div 5xy = \dfrac{15x^3 y^2}{5xy} = 3xy$

While dividing the coefficients we make use of the ordinary division rule and while dividing the powers of the variables, we make use of the laws of indices.

$$x^2 \div x^3 = \frac{x^2}{x^3} = \frac{1}{x}$$

$\dfrac{1}{x}$ is not a polynomial. **In a polynomial the variable will always have a positive exponent (index).**

$$(15x^3 + 8x^2 - 5x + 3) \div 10 = \frac{15x^3}{10} + \frac{8x^2}{10} - \frac{5x}{0} + \frac{3}{10}$$

$$= \frac{3}{2}x^3 + \frac{4}{5}x^2 - \frac{1}{2}x + \frac{3}{10}$$

$$(x^3 + 4x^2 - 9x) \div x = \frac{x^3}{x} + \frac{4x^2}{x} - \frac{9x}{x}$$

$$= x^2 + 4x - 9$$

$$(8x^5 - 9x^4 + 10x^3 + 6x^2) \div x^2$$

$$= \frac{8x^5}{x^2} + \frac{-9x^4}{x^2} + \frac{10x^3}{x^2} + \frac{6x^2}{x^2}$$

$$= 8x^3 - 9x^2 + 10x + 6.$$

$(5x^4 - 6x^3 + 10x^2 - 7x) \div 9x$

$$= \frac{5x^4}{9x} + \frac{-6x^3}{9x} + \frac{10x^2}{9x} + \frac{-7x}{9x}$$

$$= \frac{5}{9}x^3 - \frac{2}{3}x^2 + \frac{10}{9}x - \frac{7}{9}$$

$$(x^3 + 2x^2 + 3x + 4) \div x^2 = \frac{x^3}{x^2} + \frac{2x^2}{x^2} + \frac{3x}{x^2} + \frac{4}{x^2}$$

$$= x + 2 + \frac{3}{x} + \frac{4}{x^2}$$

(Not a polynomial)

If $(3x + 4)$ is divided by x^2, we do not get a polynomial. On dividing $x^3 + 2x^2 + 3x + 4$ by x^2, the quotient is $x + 2$ and the remainder is $3x + 4$. Note that the quotient and the remainder are polynomials.

Exercise 6—1 (Mental Sums)

1. Divide the polynomials given on the LHS, by each one of the monomials given on the RHS.

(a) $5x^3 - 4x^2 + 8x - 6$; $\quad 2, -2, 3, \frac{5}{2}$

(b) $5x^4 - 6x^3 + 7x^2$; $\quad 2x^2, -3x, 4, -x^2$

(c) $4x^5 - 6x^4 + 8x^3 - 10x^2$; $\quad -2x^2, +2x^2, 4x, -6$

(d) $x^6 + x^8 + x$; $\quad x, 2, -x, -2.$

2. Perform each one of the following divisions and find the quotient and the remainder.

(a) $2x^2 - 4x + 5$; $\quad 2, -2, x, 2x, -x.$

(b) $3x^4 - 2x^3 + 4x^2 - 5x + 6$; $\quad x, -x, x^2, 2x^2, x^3$

(c) $5x^4 + 10x^3 - 15x^2 - 4x - 8$; $\quad 5x^2, 2x, -5x, 2x^2, -5x^2$

(d) $3x^4 + 12x^3 - 5x - 6$; $\quad 3x^2, -4x^2, 2x^3, 4x, -2x^3$

6—2. Polynomials — Division by binomials

Note :

Divide 156 by 13.

$$\begin{array}{r|l} & 12 \\ \hline 13 & 156 \\ & 13\downarrow \\ \hline & 26 \\ & 26 \\ \hline \end{array}$$

We divide one polynomial by another as we do in ordinary division.

Example 1 :

Divide $x^2 + 5x + 6$ by $x + 3$

$$\begin{array}{r|r l} & x + 2 & \\ \hline x + 3 & x^2 + 5x + 6 & \\ & x^2 + 3x \quad \downarrow & x(x+3) \\ \hline & 2x + 6 & \\ & 2x + 6 & 2(x+3) \\ \hline & 0 & \end{array}$$

$\therefore$ $(x^2 + 5x + 6) \div (x + 3) = x + 2$

First of all write the polynomial in descending order.

1. The first term of the dividend is x^2

   The first term of the divisor is x

   The first term of the quotient $= \frac{x^2}{x} = x$

2. Multiply $(x + 3)$ by x and after writing the various terms under the like terms of the given polynomial (ie., under $x^2 + 5x$) subtract the same.

3. To this difference $2x$ add the next term of the polynomial and keep it as the new dividend.

4. The next term of the quotient is $\frac{2x}{x} = 2$.

5. Multiply $x + 3$ by 2 and write the product under the new dividend and subtract.

6. This can be repeated as long as we need.

Example 2 :

Divide $8y^3 - 1$ by $2y - 1$

$$\begin{array}{r|l|l}
 & 4y^2 + 2y + 1 & \\
\hline
2y - 1 & 8y^3 + 0y^2 + 0y - 1 & \\
 & 8y^3 - 4y^2 & 4y^2(2y - 1) \\
\hline
 & \quad 4y^2 + 0y & \\
 & \quad 4y^2 - 2y & 2y(2y - 1) \\
\hline
 & \qquad 2y - 1 & \\
 & \qquad 2y - 1 & 1(2y - 1) \\
\hline
 & \qquad\quad 0 &
\end{array}$$

$\therefore (8y^3 - 1) \div (2y - 1) = 4y^2 + 2y + 1$

Example 3 :

Divide $27x^3 - 54x^2y + 36xy^2 - 8y^3$ by $3x - 2y$.

$$\begin{array}{r|l|l}
 & 9x^2 - 12xy + 4y^2 & \\
\hline
3x - 2y & 27x^3 - 54x^2y + 36xy^2 - 8y^3 & \\
 & 27x^3 - 18x^2y & 9x^2(3x - 2y) \\
\hline
 & \quad -36x^2y + 36xy^2 & \\
 & \quad -36x^2y + 24xy^2 & -12xy(3x - 2y) \\
\hline
 & \qquad +12xy^2 - 8y^3 & \\
 & \qquad 12xy^2 - 8y^3 & 4y^2(3x - 2y) \\
\hline
 & \qquad\quad 0 &
\end{array}$$

$\therefore (27x^3 - 54x^2y + 36xy^2 - 8y^3) \div (3x - 2y)$
$= 9x^2 - 12xy + 4y^2$

Exercise 6—2

Compute and find the quotient:

(1) $(x^2 + 4x + 3) \div (x + 3)$

(2) $(x^2 + 11x + 18) \div (x + 9)$

(3) $(x^2 - 12x + 27) \div (x - 3)$

(4) $(x^2 - 10x + 25) \div (x - 5)$

(5) $(x^2 + 11x - 42) \div (x - 3)$

(6) $(x^2 - 6x - 27) \div (x - 9)$

(7) $(x^2 + 4xy + 3y^2) \div (x + 3y)$

(8) $(x^2 + 12xy + 27y^2) \div (x + 9y)$

(9) $(a^2 - 12ab + 27b^2) \div (a - 3b)$

(10) $(m^2 - 10mn + 25n^2) \div (m - 5n)$

(11) $(p^2 + 11pq - 42q^2) \div (p - 3q)$

(12) $(k^2 - 6kl - 27l^2) \div (k - 9l)$

(13) $(27a^3 + 8) \div (3a + 2)$

(14) $(16x^4 - 81) \div (2x - 3)$

(15) $(64m^2 - 27) \div (4m - 3)$

(16) $(6x^3 - 23x^2y + 24xy^2 - 10y^3) \div (2x - 5y)$

(17) $(18x^3 - 17xy^2 + 20y^3) \div (3x + 4y)$

(18) $(10a^3 + 19a^2b - 9b^3) \div (5a - 3b)$

(19) $(6x^3 - 23x^2y + 24xy^2 - 10y^3) \div (2x - 5y)$

(20) $(18x^3 - 17xy^2 + 20y^3) \div (3x + 4y)$

Answers

(1) $x + 1$ (2) $x + 2$ (3) $x - 9$ (4) $x - 5$
(5) $x + 14$ (6) $x + 3$ (7) $x + y$ (8) $x + 3y$
(9) $a - 9b$ (10) $m - 5n$ (11) $p + 14q$ (12) $k + 3l$
(13) $9a^2 - 6a + 4$ (14) $8x^3 + 12x^2 + 18x + 27$
(15) $6m^2 + 12m + 9$ (16) $3x^2 - 4xy + 2y^2$
(17) $6x^2 - 8xy + 5y^2$.

6-3. Polynomials — Division by Trinomials

Look at the given division. Find out the similarity between this and the following division of one polynomial by another.

```
        231
121 ) 27951
      242
      ---
       375
       363
       ---
        121
        121
        ---
          0
```

$$(2x^4 + 7x^3y + 9x^2y^2 + 5xy^3 + y^4) \div (x^2 + 2xy + y^2)$$

$$\begin{array}{r|l|l}
 & \quad\quad\quad 2x^2 \quad + 3xy \quad + y^2 & \\
x^2 + 2xy + y^2 & 2x^4 + 7x^3y + 9x^2y^2 + 5xy^3 + y^4 & 2x^2\,(x^2 + 2xy + y^2) \\
 & \underline{2x^4 + 4x^3y + 2x^2y^2} & \\
 & \quad\quad 3x^3y + 7x^2y^2 + 5xy^3 & 3xy\,(x^2 + 2xy + y^2) \\
 & \quad\quad \underline{3x^3y + 6x^2y^2 + 3xy^3} & \\
 & \quad\quad\quad\quad x^2y^2 + 2xy^3 + y^4 & y^2\,(x^2 + 2xy + y^2) \\
 & \quad\quad\quad\quad \underline{x^2y^2 + 2xy^3 + y^4} & \\
 & \quad\quad\quad\quad\quad\quad\quad\quad 0 &
\end{array}$$

Exercise 6—3

Find the quotient :

(1) $(2x^3 + 5x^2 + 4x + 1) \div (x^2 + 2x + 1)$

(2) $(2x^3 - 5x^2 + 4x - 1) \div (x^2 - 2x + 1)$

(3) $(x^6 - x^4 - x^2 + 1) \div (x^2 + 2x + 1)$

(4) $(4x^3 + 4x^2 - 5x - 3) \div (2x^2 - x - 1)$

(5) $(24x^4 + 146x^3 + 409x^2 \div 326x + 120) \div (2x^2 + 7x + 6)$

(6) $(2x^3 - x^2y - 2xy^2 + y^3) \div (2x^2 - 3xy + y^2)$

(7) $(2x^4 + 7x^3y + 13x^2y^2 + 11xy^3 + 3y^4) \div (x^2 + 2xy + 3y^2)$

(8) $(6x^4 - x^3y - 4x^2y^2 + 22xy^3 - 8y^4) \div (3x^2 + 4xy - 2y^2)$

(9) $(5x^4 + 3x^3y + 5xy^3 - 3y^4) \div (x^2 - xy + y^2)$

(10) $(a^4 + a^2b^2 + b^4) \div (a^2 + ab + b^2)$

Answers

(1) $2x + 1$ (2) $2x - 1$ (3) $x^4 - 2x^3 + 2x^2 - 2x + 1$
(4) $2x + 3$ (7) $2x^2 + 3xy + y^2$ (10) $a^2 - ab + b^2$.

6—4. Polynomials — Division — Finding the quotient and the remainder

So far we studied about the division of one polynomial by another where the remainder was zero. Now by dividing polynomials, we shall find the quotient as well as the remainder. The process is the same as in ordinary division.

Example:

$$
\begin{array}{r|l|l}
 & x^2 + 2x + 3 & \\
\hline
2x^2 + 3x + 1 & 2x^4 + 7x^3 + 13x^2 + 15x + 6 & \\
 & 2x^4 + 3x^3 + x^2 & x^2(2x^2 + 3x + 1) \\
\cline{2-2}
 & 4x^3 + 12x^2 + 15x & \\
 & 4x^3 + 6x^2 + 2x & 2x(2x^2 + 3x + 1) \\
\cline{2-2}
 & 6x^2 + 13x + 6 & \\
 & 6x^2 + 9x + 3 & 3(2x^2 + 3x + 1) \\
\cline{2-2}
 & 4x + 3 &
\end{array}
$$

Quotient: $x^2 + 2x + 3$; Remainder: $4x + 3$

The quotient and the remainder are polynomials. See that the degree of the remainder is less than that of the divisor. Can you find out the reason for this.

Exercise 6—4

Find the quotient and the remainder:

(1) $(2x^3 - 5x^2 + 6x + 5) \div (x^2 - 2x + 1)$

(2) $(24x^4 + 146x^3 + 409x^2 + 300x + 100) \div (2x^2 + 7x + 6)$

(3) $(6x^4 - x^3y - 4x^2y^2 + 26xy^3 - 4y^4) \div (3x^2 + 4xy - 2y^2)$

(4) $(5x^4 + 3x^3y + 3y^4) \div (x^2 - xy + y^2)$

(5) $x^5 \div (x^2 + 2x + 3)$

Answers

(1) Quotient: $(2x - 1)$, Remainder: $(2x + 6)$

(4) Quotient: $(5x^2 + 8xy + 3y^2)$, Remainder: $(-5xy^3)$

(5) Quotient: $(x^3 - 2x^2 + x + 4)$, Remainder: $(-11x - 12)$

7—1. Identities — Revision — Expansion of $(a \pm b)^2$

Let us now recall what all we studied about the identities in the VIII Standard.

$$(a + b)^2 = a^2 + 2ab + b^2$$
$$(a - b)^2 = a^2 - 2ab + b^2$$

Examples:

$$\underset{\downarrow}{a} \quad \underset{\downarrow}{b} \qquad \underset{\downarrow}{a^2} + 2 \times \underset{\downarrow}{a} \times \underset{\downarrow}{b} + \underset{\downarrow}{b^2}$$
$$(2x + 3y)^2 = (2x)^2 + 2 \times 2x \times 3y + (3y)^2$$
$$= 4x^2 + 12xy + 9y^2$$

$$\underset{\downarrow}{a} \quad \underset{\downarrow}{b} \qquad \underset{\downarrow}{a^2} - 2 \times \underset{\downarrow}{a} \times \underset{\downarrow}{b} + \underset{\downarrow}{b^2}$$
$$(3x - 4y)^2 = (3x)^2 - 2 \times 3x \times 4y + (4y)^2$$
$$= 9x^2 - 24xy + 16y^2$$

$$105^2 = (100 + 5)^2$$
$$= 100^2 + 2 \times 100 \times 5 + 5^2$$
$$= 10000 + 1000 + 25 = 11025$$

$$94^2 = (100 - 6)^2$$
$$= 100^2 - 2 \times 100 \times 6 + 6^2$$
$$= 10000 - 1200 + 36 = 8836$$

Exercise 7—1

I. Find the square of the following expressions (Mental sums)

(a)

1. $a + b$	2. $p + q$
3. $l + m$	4. $x + y$
5. $m + n$	6. $a + 2$
7. $p + 3$	8. $l + 4$
9. $x + 5$	10. $m + 6$
11. $a + 2b$	12. $p + 3q$
13. $l + 4m$	14. $x + 5y$
15. $m + 6n$	16. $a + 3b$

17. $4p + q$ 18. $5l + m$
19. $6x + y$ 20. $7m + n$
21. $3a + 2b$ 22. $4p + 5q$
23. $5l + 3m$ 24. $6x + 7y$

(b) 1. $a - b$ 2. $p - q$
3. $l - m$ 4. $x - y$
5. $m - n$ 6. $a - 2$
7. $p - 3$ 8. $l - 4$
9. $x - 5$ 10. $m - 6$
11. $a - 2b$ 12. $p - 3q$
13. $l - 4m$ 14. $x - 5y$
15. $m - 6n$ 16. $a - 3b$
17. $4p - q$ 18. $5l - m$
19. $6x - y$ 20. $7m - n$
21. $3a - 2b$ 22. $4p - 3q$
23. $5l - 3m$ 24. $6x - 7y$

II. Simplify using the proper formula:

(a) $49 + 2 \times 3 \times 7 + 9$
(b) $225 + 2 \times 15 \times 5 + 25$
(c) $625 + 2 \times 25 \times 15 + 225$
(d) $169 - 2 \times 13 \times 3 + 9$
(e) $144 - 2 \times 12 \times 2 + 4$
(f) $77 \times 77 + 2 \times 77 \times 23 + 23 \times 23$
(g) $95 \times 95 + 2 \times 95 \times 5 + 5 \times 5$
(h) $65 \times 65 - 2 \times 65 \times 15 + 15 \times 15$
(i) $112 \times 112 - 2 \times 112 \times 12 + 12 \times 12$
(j) $88 \times 88 - 2 \times 88 \times 88 + 88 + 88$

III. Find the values of the following with the help of proper formulae.

(1) 101^2 (2) 53^2 (3) 22^2 (4) 44^2 (5) 61^2
(6) 99^2 (7) 47^2 (8) 38^2 (9) 36^2 (10) 59^2

Answers

II. (a) 100 (b) 400 (c) 1600 (d) 100
(e) 100 (f) 10000 (g) 10000 (h) 2500
(i) 10000 (j) 0.

7—2. Identities — Expansion of $(a + b + c)^2$

$(a + b + c)^2 = (a + b + c) \times (a + b + c)$

$$\begin{array}{l} a + b + c \\ a + b + c \\ \hline a^2 + ab + ac \\ \quad + ba \quad + b^2 + bc \\ \quad\quad + ca \quad + cb + c^2 \\ \hline a^2 + 2ab + 2ca + b^2 + 2bc + c^2 \\ \hline \end{array}$$

$$\boxed{(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca}$$

The same result can be obtained by making use of the identity $(a + b)^2 = a^2 + 2ab + b^2$

Let $b + c = x$, $a + b + c = a + (b + c) = a + x$

$$\begin{aligned} \therefore \quad (a + b + c)^2 &= (a + x)^2 = a^2 + 2ax + x^2 \\ &= a^2 + 2a(b + c) + (b + c)^2 \\ &= a^2 + 2ab + 2ac + b^2 + 2bc + c^2 \\ &= a^2 + b^2 + c^2 + 2ab + 2bc + 2ca \end{aligned}$$

Examples :

1. $(2a + 3b + 5c)^2$

$= (2a)^2 + (3b)^2 + (5c)^2 + 2 \times 2a \times 3b + 2 \times 3b \times 5c$
$+ 2 \times 5c \times 2a$

$= 4a^2 + 9b^2 + 25c^2 + 12ab + 30bc + 20ca$

2. $(3a - 4b + 2c)^2$

$= (3a)^2 + (-4b)^2 + (2c)^2 + 2 \times (3a)(-4b)$
$+ 2 \times (-4b)(2c) + 2 \times (2c)(3a)$

$= 9a^2 + 16b^2 + 4c^2 - 24ab - 16bc + 12ca$

$[\because (-4b)^2 = -4b \times -4b = 16b^2]$

3. $(5x - 3y - 4z)^2$

$= (5x)^2 + (-3y)^2 + (-4z)^2 + 2 \times (5x)(-3y)$
$+ 2 \times (-3y)(-4z) + 2(-4z)(5x)$

$= 25x^2 + 9y^2 + 16z^2 - 30xy + 24yz - 40zx$

4. $123^2 = (100 + 20 + 3)^2$

= 10000	(100^2)
400	(20^2)
9	(3^2)
4000	$(2 \times 100 \times 20)$
120	$(2 \times 20 \times 3)$
600	$(2 \times 3 \times 100)$
15129	

Exercise 7—2

I. Expand (Mental sums)

1. $(a + b + c)^2$
2. $(p + q + r)^2$
3. $(l + m + n)^2$
4. $(x + y + z)^2$
5. $(a + b - c)^2$
6. $(p + q - r)^2$
7. $(l + m - n)^2$
8. $(x + y - z)^2$
9. $(a - b + c)^2$
10. $(p - q + r)^2$
11. $(l - m + n)^2$
12. $(x - y + z)^2$
13. $(a - b - c)^2$
14. $(p - q - r)^2$
15. $(l - m - n)^2$
16. $(x - y - z)^2$
17. $(2a + 3b + 4c)^2$
18. $(3p + 4q + 2r)^2$
19. $(2l + 3m + 7n)^2$
20. $(5x + 2y + 4z)^2$
21. $(2a - 3b + 4c)^2$
22. $(3p - 4q - 2r)^2$
23. $(2l + 3m - 7n)^2$
24. $(5x - 2y - 4z)^2$

II. Find the values of the following making use of the formulae:

1. 111^2 2. 125^2 3. 234^2 4. 425^2 5. 542^2.

7—3. Identities — Expansion of $(a + b)^3$, $(a - b)^3$

Remember that $x \times x^2 = x^3$

$$(a + b)^2 + a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$\begin{aligned}
(a + b)^3 &= (a + b)(a + b)^2 \\
&= (a + b)(a^2 + 2ab + b^2) \\
&= a(a^2 + 2ab + b^2) + b(a^2 + 2ab + b^2) \\
&= a^3 + 2a^2b + ab^2 + a^2b + 2ab^2 + b^3 \\
&= a^3 + 3a^2b + 3ab^2 + b^3
\end{aligned}$$

$$\begin{aligned}
(a - b)^3 &= (a - b)(a - b)^2 \\
&= (a - b)(a^2 - 2ab + b^2) \\
&= a(a^2 - 2ab + b^2) - b(a^2 - 2ab + b^2) \\
&= a^3 - 2a^2b + ab^2 - a^2b + 2ab^2 - b^3 \\
&= a^3 - 3a^2b + 3ab^2 - b^3
\end{aligned}$$

Note : The expressions are in descending powers of 'a' and ascending powers of 'b'.

Expression	Expansion	coefficients
$(a + b)$	$a + b$	1, 1
$(a + b)^2$	$a^2 + 2ab + b^2$	1, 2, 1
$(a + b)^3$	$a^3 + 3a^2b + 3ab^2 + b^3$	1, 3, 3, 1
$(a - b)$	$a - b$	1, — 1
$(a - b)^2$	$a^2 - 2ab + b^2$	1, — 2, 1
$(a - b)^3$	$a^3 - 3a^2b + 3ab^2 - b^3$	1, — 3, 3, — 1

Make a comparative as well as an individual study of the coefficients of the pairs of identities $(a + b)^2$, $(a - b)^2$; $(a + b)^3$, $(a - b)^3$. Make a note of the signs of the coefficients also.

Investigate about the similarities and dissimilarities present.

Examples :

$$\underset{a}{\downarrow} \quad \underset{b}{\downarrow} \qquad a^3 + 3a^2 b + 3ab^2 + b^3$$

1. $(2x + 5y)^3 = (2x)^3 + 3(2x)^2(5y) + 3(2x)(5y)^2 + (5y)^3$

$= 8x^3 + 3(4x^2)(5y) + 3(2x)(25y^2) + 125y^3$

$= 8x^3 + 60x^2y + 150xy^2 + 125y^3$

2. $(3x - 2y)^3 = (3x)^3 - 3(3x)^2(2y) + 3(3x)(2y)^2 - (2y)^3$

$= 27x^3 - 3(9x^2)(2y) + 3(3x)(4y^2) - 8y^3$

$= 27x^3 - 54x^2y + 36xy^2 - 8y^3$

3. $52^3 = (50 + 2)^3$

$= 50^3 + 3 \times 50^2 \times 2 + 3 \times 50 \times 2^2 + 2^3$

$= 125000 + 3 \times 2500 \times 2 + 3 \times 50 \times 4 + 8$

$= 125000 + 15000 + 600 + 8$

$= 140608$

Exercise 7—3

I Find the cubes of the expressions in (a) and (b) in Ex. 7—1.

II Simplify with the help of formulae:

1. $7^3 + 3 \times 7^2 \times 3 + 3 \times 7 \times 3^2 + 3^3$
2. $28^3 + 3 \times 28^2 \times 2 + 3 \times 28 + 2^2 + 2^3$
3. $88^3 + 3 \times 88^2 \times 12 + 3 \times 88 \times 12^2 + 12^3$
4. $15^3 - 3 \times 15^2 \times 5 + 3 \times 15 \times 5^2 - 5^3$
5. $62^3 - 3 \times 62^2 \times 12 + 3 \times 62 \times 12^2 - 12^3$
6. $112^3 - 3 \times 112^2 \times 12 + 3 \times 112 \times 12^2 - 12^3$

III Find the values making use of the formulae.

1. 22^3 2. 101^3 3. 48^3 4. 99^3 5. 33^3

IV If the relationship between the coefficients of the expansions $(a + b)^2$ and $(a + b)^3$ holds good for the coefficients of the expansion of $(a + b)^4$, try to find out the coefficients of the expansion of $(a + b)^4$.

Answers

II (1) 1000 (2) 27000 (3) 1000000

(4) 1000 (5) 125000 (6) 1000000

IV 1, 4, 6, 4, 1.

7—4. Identities — Expansion of $(x + a)(x + b)$ — Revision

In the VIII standard you have learnt that

$$(x + a)(x + b) = x^2 + (a + b)x + ab$$

From this you can understand that

$$(x + a)(x - b) = x^2 + (a - b)x - ab$$

$$(x - a)(x + b) = x^2 + (b - a)x - ab$$

$$(x - a)(x - b) = x^2 - (a + b)x + ab$$

Examples :

1. $(x + 3)(x + 5) = x^2 + (3 + 5)x + 3 \times 5$
   $= x^2 + 8x + 15$
2. $(x + 5)(x - 3) = x^2 + (5 - 3)x - 5 \times 3$
   $= x^2 + 2x - 15$
3. $(x - 5)(x + 3) = x^2 + (-5 + 3)x - 5 \times 3$
   $= x^2 - 2x - 15$
4. $(x - 5)(x - 3) = x^2 - (3 + 5)x + 5 \times 3$
   $= x^2 - 8x + 15$

[Examine the coefficients of the middle terms and the coefficients of the last terms.]

5. $(x + 3y)(x + 5y) = x^2 + (3y + 5y)x + 3y \times 5y$
$= x^2 + 8xy + 15y^2$

6. $(x - 2y)(x + 7y) = x^2 + (7 - 2)xy - 2y \times 7y$
$= x^2 + 5xy - 14y^2$

7. $(2x + 7)(2x - 5) = (2x)^2 + (7 - 5)2x - 7 \times 5$
$= 4x^2 + 4x - 35$

8. $(3x - 4)(3x - 2) = (3x)^2 - (4 + 2)3x + 4 \times 2$
$= 9x^2 - 18x + 8$

9. $23 \times 27 = (20 + 3)(20 + 7)$
$= 20^2 + (3 + 7)20 + 3 \times 7$
$= 400 + 200 + 21 = 621$

10. $47 \times 38 = (40 + 7)(40 - 2)$
$= 40^2 + (7 - 2)40 - 7 \times 2$
$= 1600 + 200 - 14 = 1786$

Exercise 7—4

I Expand (Mental sums)

1. $(x + 3)(x + 2)$
2. $(a + 4)(a + 5)$
3. $(m + 5)(m + 6)$
4. $(p + 6)(p + 3)$
5. $(l + 8)(l + 9)$
6. $(x + 3y)(x + 2y)$
7. $(a + 4b)(a + 5b)$
8. $(m + 5n)(m + 6n)$
9. $(p + 6q)(p + 3q)$
10. $(l + 8m)(l + 9m)$
11. $(x + 3)(x - 2)$
12. $(a + 4)(a - 5)$
13. $(a + 5)(a - 6)$
14. $(p + 6)(p - 3)$
15. $(l + 8)(l - 9)$
16. $(x + 3y)(x - 2y)$
17. $(a + 4b)(a - 5b)$
18. $(m + 5n)(m - 6n)$
19. $(p + 6q)(p - 3q)$
20. $(l + 8m)(l - 9m)$
21. $(x - 3)(x + 2)$
22. $(a - 4)(a - 5)$
23. $(m - 5)(m - 6)$
24. $(p - 6)(p + 3)$
25. $(l - 8)(l + 9)$
26. $(x - 3y)(x + 2y)$
27. $(a - 4b)(a + 5b)$
28. $(m - 5n)(m + 6n)$
29. $(p - 6q)(p + 3q)$
30. $(l - 8m)(l + 9m)$

31. $(x - 3)(x - 2)$
32. $(a - 4)(a - 5)$
33 $(m - 5)(m - 6)$
34. $(p - 6)(p - 3)$
35. $(l - 8)(l - 9)$
36. $(x - 3y)(x - 2y)$
37. $(a - 4b)(a - 5b)$
38. $(m - 5n)(m - 6n)$
39. $(p - 6q)(p - 3q)$
40. $(l - 8m)(l - 9m)$

II Find the expansions (First find the expansions. Later try to do them as mental sums.)

1. $(2x + 3)(2x + 5)$
2. $(3a + 4)(3a + 5)$
3. $(4m + 5n)(4m + 7n)$
4. $(7p + 6q)(7p + 9q)$
5. $(5l + 8m)(5l + 9m)$
6. $(2x + 3)(2x - 5)$
7. $(3a + 4)(3a - 5)$
8. $(4m + 5n)(4m - 3n)$
9. $(7p + 6q)(7p - 9q)$
10. $(5l + 8m)(5l - 9m)$
11. $(2x - 3)(2x + 5)$
12. $(3a - 4)(3a + 5)$
13 $(4m + 5n)(4m - 7n)$
14 $(7p - 6q)(7p + 9q)$
15 $(5l - 8m)(5l + 9m)$
16 $(2x - 3)(2x - 5)$
17. $(3a - 4)(3a - 5)$
18. $(4m - 5n)(4m - 7n)$
19 $(7p - 6q)(7p - 9q)$
20. $(5l - 8m)(5l - 9m)$

III Find the expansions of $(a + b)^2$, $(a - b)^2$ and $(a + b)(a - b)$ with the help of the formula seen above.

IV Find the products making use of the formulae. (Find as many products as possible mentally.)

(1) 34×36 (2) 72×78 (3) 42×43

(4) 46×53 (5) 76×83 (6) 94×102

7—5. Identities — Expansion of $(x + a)(x + b)(x + c)$

Recall : $(x + a)(x + b) = x^2 + (a + b)x + ab;$

$$a(b + c) = ab + ac$$

$$(x + a)(x + b)(x + c) = (x + a)[(x + b)(x + c)]$$

$$= (x + a)[x^2 + (b + c)x + bc]$$

$= x[x^2 + (b + c)x + bc] + a[x^2 + (b + c)x + bc]$

$= x^3 + (b + c)x^2 + bcx + ax^2 + a(b + c)x + abc$

$= x^3 + (a + b + c)x^2 + (ab + bc + ca)x + abc.$

Similarly we can find that

$(x - a)(x - b)(x - c) = x^3 + (-a - b - c)x^2$

$+ [(-a)(-b) + (-b)(-c) + (-c)(-a)]x$

$+ (-a)(-b)(-c)$

$= x^3 - (a + b + c)x^2 + (ab + bc + ca)x - abc$

Examples :

1. $(x + 4)(x + 5)(x + 3)$

$= x^3 + (4 + 5 + 3)x^2 + (20 + 15 + 12)x + 60$

$= x^3 + 12x^2 + 47x + 60$

2. $(x - 4)(x - 5)(x - 3)$

$= x^3 - (4 + 5 + 3)x^2 + (20 + 15 + 12)x - 60$

$= x^3 - 12x^2 + 47x - 60$

3. $(x + 4)(x + 5)(x - 3)$

$= x^3 + (4 + 5 - 3)x^2 + (20 - 15 - 12)x - 60$

$= x^3 + 6x^2 - 7x - 60$

4 $(x - 4)(x + 5)(x - 3)$

$= x^3 + (-4 + 5 - 3)x^2 + (-20 - 15 + 12)x + 60$

$= x^3 - 2x^2 - 23x + 60$

5. $(2x + 5)(2x - 3)(2x + 4)$

$= (2x)^3 + (5 - 3 + 4)(2x)^2 + (-15 - 12 + 20)(2x) - 60$

$= 8x^3 + 6(4x^2) + (-7)(2x) - 60$

$= 8x^3 + 24x^2 - 14x - 60.$

6. Find the coefficients of x^2, x in the expansion of $(4x - 3)\ (4x + 7)\ (4x - 2)$

Term containing x^2

$(-3 + 7 - 2)(4x)^2 = 2 \times 16x^2 = 32x^2$

$\therefore$ **Coefficient of $x^2 = 32$.**

Term containing x

$= (-21 - 14 + 6)(4x) = -29 \times 4x = -116x$

$\therefore$ **Coefficient of $x = -116$.**

Exercise 7—5

I Expand: [Try to do mentally as many as possible]

1. $(x+3)(x+6)(x+2)$
2. $(p+4)(p+5)(p+6)$
3. $(m+1)(m+2)(m+3)$
4. $(a+2)(a+3)(a+4)$
5. $(l+5)(l+2)(l+3)$
6. $(x+3)(x+6)(x-2)$
7. $(p+4)(p-5)(p+6)$
8. $(m-1)(m+2)(m+3)$
9. $(a+2)(a-3)(a+4)$
10. $(l+5)(l+2)(l-3)$
11. $(x+3)(x-6)(x+2)$
12. $(p-4)(p-5)(p+6)$
13. $(m-1)(m+2)(m-3)$
14. $(a+2)(a-3)(a-4)$
15. $(l-5)(l+2)(l-3)$
16. $(x-3)(x-6)(x-2)$
17. $(p-4)(p-5)(p-6)$
18. $(m-1)(m-2)(m-3)$
19. $(a-2)(a-3)(a-4)$
20. $(l-5)(l-2)(l-3)$

II Expand :

1. $(2x+3)(2x+5)(2x+7)$
2. $(3p+4)(3p+5)(3p+2)$
3. $(2m+1)(2m+3)(2m+5)$
4. $(5a+2)(5a+3)(5a+4)$
5. $(4l+5)(4l+7)(4l+3)$
6. $(2x+3)(2x-5)(2x+7)$
7. $(3p-4)(3p+5)(3p+2)$
8. $(2m-1)(2m+3)(2m-5)$
9. $(5a+2)(5a-3)(5a-4)$
10. $(4l-5)(4l-7)(4l-3)$

III Find the coefficients of x^2, x alone in the following expansions :

1. $(\ x+5)(\ x-2)(\ x+3)$
2. $(\ x+6)(\ x-2)(\ x-5)$

3. $(x - 6)(x - 2)(x - 3)$
4. $(x + 5)(x - 2)(x - 3)$
5. $(x + 2)(x - 3)(x - 6)$
6. $(2x + 3)(2x + 5)(2x + 7)$
7. $(3x + 4)(3x + 2)(3x + 7)$
8. $(4x - 2)(4x + 3)(4x - 1)$
9. $(5x - 3)(5x - 1)(5x + 2)$
10. $(10x - 9)(10x - 3)(10x - 1)$

Answers

I (1) $x^3 + 11x^2 + 36x + 36$
(7) $p^3 + 5p^2 - 26p - 120$
(16) $x^3 - 11x^2 + 36x - 36$

II (1) $8x^3 + 60x^2 + 142x + 105$
(6) $8x^3 + 20x^2 - 58x - 105$
(8) $8m^3 - 12m^2 - 26m + 15$
(10) $64l^3 - 240l^2 + 284l - 105$

III (1) $6, -1$ (2) $-1, -32$ (3) $-11, 36$
(6) $60, 142$ (8) $0, -28$ (9) $-50. -25$
(10) $-1300, 390$.

7—6. Identities: $(a + b)(a^2 - ab + b^2) = a^3 + b^3$
$(a - b)(a^2 + ab + b^2) = a^3 - b^3$

$$(a + b)(a^2 - ab + b^2)$$
$$= a(a^2 - ab + b^2) + b(a^2 - ab + b^2)$$
$$= a^3 - a^2b + ab^2 + a^2b - ab^2 + b^3$$
$$= a^3 + b^3$$

$$(a - b)(a^2 + ab + b^2)$$
$$= a(a^2 + ab + b^2) - b(a^2 + ab + b^2)$$
$$= a^3 + a^2b + ab^2 - a^2b - ab^2 - b^3$$
$$= a^3 - b^3$$

Later we can learn that the above indentities can also be derived from the expansions of $(a + b)^3$, $(a - b)^3$.

Examples :

(1) $(a + 2)(a^2 - 2a + 4) = a^3 + 2^3 = a^3 + 8$

(2) $(2a + 3)(4a^2 - 6a + 9) = (2a)^3 + 3^3 = 8a^3 + 27$

(3) $(a - 3)(a^2 + 3a + 9) = a^3 - 3^3 = a^3 - 27$

(4) $(3a - 5)(9a^2 + 15a + 25) = (3a)^3 - 5^3$
$= 27a^3 - 125$

Exercise 7—6

I See whether the following are in the form $(a - b)(a^2 - ab + b^2)$ or $(a - b)(a^2 + ab + b^2)$. Find their expansions.

(1) $(p + q)(p^2 - pq + q^2)$

(2) $(l + m)(l^2 + lm + m^2)$

(3) $(x - y)(x^2 + xy + y^2)$

(4) $(a + 2b)(a^2 + 2ab + b^2)$

(5) $(a - 2b)(a^2 + 2ab + 2b^2)$

(6) $(a - 2b)(a^2 + 2ab + 4b^2)$

(7) $(2a + b)(4a^2 - 2ab + b^2)$

(8) $(3a - 4)(9a^2 - 12a + 16)$

(9) $(2a - 5)(4a^2 + 10a + 25)$

(10) $(3a + 2b)(9a^2 - 6ab + 4b^2)$

II In the above problems rewrite those which are not in the form $(a + b)(a^2 - ab + b^2)$ or $(a - b)(a^2 + ab + b^2)$ in the said form and write the expansion.

III Fill in the blanks :

(1) $(x + y)(\qquad\qquad)\quad x^3 + y^3$

(2) $(\qquad)(a^2 - 2ab \div 4b^2) = a^3 + 8b^3$

(3) $(p - 2q)(\qquad\qquad) = p^3 - 8q^3$

(4) $(2l - 3)(\qquad\qquad) = 8l^3 - 27$

(5) $(\qquad)(\qquad\qquad) = 27x^3 - 64$

Answers

I (1) $p^3 + q^3$ (2) Not in the form (3) $x^3 - y^3$

(4) Not in the form (5) Not in the form (6) $a^3 - 8b^3$

(7) $8a^3 + b^3$ (8) Not in the form (9) $27a^3 + 8b^3$

III (1) $x^2 - xy + y^2$ (2) $a + 2b$

(3) $p^2 + 2pq + 4q^2$ (4) $4l^2 + 6l + 9$

(5) $(3x - 4)(9x^2 + 12x + 16)$

7—7. Identities — Applications (a)

You know that all the identities studied so far are polynomials. Since the identities take up values from the real number system, they will hold good for rational expressions also under certain conditions.

Simplify :

1. $\left(x + \frac{1}{x}\right)^2 = x^2 + 2 \times x \times \frac{1}{x} + \frac{1}{x^2}$ $(x \neq 0)$

$= x^2 + 2 + \frac{1}{x^2}$

2. $\left(\frac{a}{b} + \frac{b}{a}\right)^2 = \left(\frac{a}{b}\right)^2 + 2.\frac{a}{b}.\frac{b}{a} + \left(\frac{b}{a}\right)^2$

$[a, b \neq 0]$

$= \frac{a^2}{b^2} + 2 + \frac{b^2}{a^2}$

3. $\left(x + \frac{1}{x}\right)^2 = x^3 + 3x^2\frac{1}{x} + 3.x.\frac{1}{x^2} + \frac{1}{x^3}$

$[x \neq 0]$

$= x^3 + 3x + \frac{3}{x} + \frac{1}{x^3}$

4. If $a + \frac{1}{a} = 3$, find the values of $a^2 + \frac{1}{a^2}$ and $a^3 + \frac{1}{a^3}$

(i) $a + \frac{1}{a} = 3; \left(a + \frac{1}{a}\right)^2 = 3^2$

$$a^2 + 2.\ a.\ \frac{1}{a} + \frac{1}{a^2} = 9$$

$$a^2 + 2 + \frac{1}{a^2} = 9; \quad a^2 + \frac{1}{a^2} = 9 - 2 = 7$$

(ii) $\left(a + \frac{1}{a}\right)^3 = 3^3$

$$a^3 + 3.\ a^2.\ \frac{1}{a} + 3.\ a.\ \frac{1}{a^2} + \frac{1}{a^3} = 27$$

$$a^3 + 3a + \frac{3}{a} + \frac{1}{a^3} = 27$$

$$a^3 + \frac{1}{a^3} + 3\left(a + \frac{1}{a}\right) = 27$$

$$a^3 + \frac{1}{a^3} + 3 \times 3 = 27$$

$$a^3 + \frac{1}{a^3} = 27 - 9 = 18$$

Exercise 7—7

I Find the squares and cubes of the following:

(1) $p + \frac{1}{p}$ (2) $m + \frac{1}{m}$ (3) $l + \frac{1}{l}$

(4) $p - \frac{1}{p}$ (5) $m - \frac{1}{m}$ (6) $l - \frac{1}{l}$

(7) $p + \frac{2}{p}$ (8) $m + \frac{3}{m}$ (9) $l + \frac{4}{l}$

(10) $p - \frac{5}{p}$ (11) $m - \frac{6}{m}$ (12) $l - \frac{9}{l}$

(13) $2p + \frac{1}{p}$ (14) $3m + \frac{1}{m}$ (15) $4l + \frac{1}{l}$

(16) $5p - \frac{1}{p}$ (17) $3m - \frac{1}{m}$ (18) $9l - \frac{1}{l}$

(19) $2p + \frac{3}{p}$ (20) $3m + \frac{4}{m}$ (21) $4l + \frac{5}{l}$

(22) $5p - \frac{2}{p}$ (23) $6m - \frac{7}{m}$ (24) $9l - \frac{8}{l}$

(25) $\frac{x}{y} + \frac{y}{x}$ (26) $\frac{m}{n} + \frac{n}{m}$ (27) $\frac{l}{m} + \frac{m}{l}$

(28) $\frac{p}{q} + \frac{q}{p}$ (29) $\frac{m}{n} - \frac{n}{m}$ (30) $\frac{a}{b} - \frac{b}{a}$

II If $x + \frac{1}{x}$ is defined on the set {1, 2, — 1, 5, — 2} find the values of $x^2 + \frac{1}{x^2}$, $x^3 + \frac{1}{x^3}$.

III If $p - \frac{1}{p}$ is defined on the set {2, 3, 4, — 2, — 3} find the values of $p^2 + \frac{1}{p^2}$, $p^3 - \frac{1}{p^3}$.

Answers

I (1) $p^2 + \frac{1}{p^2} + 2$; $p^3 + \frac{1}{p^3} + 3p + \frac{3}{p}$

(4) $p^2 + \frac{1}{p^2} - 2$; $p^3 - \frac{1}{p^3} - 3p + \frac{3}{p}$

(7) $p^2 + \frac{4}{p^2} + 4$; $p^3 + \frac{8}{p^3} + 6p + \frac{12}{p}$

(10) $p^2 + \frac{25}{p^2} - 10;\ p^3 - \frac{125}{p^3} + 15p - \frac{75}{p}$

(13) $4p^2 + \frac{1}{p^2} + 4;\ 8p^3 + \frac{1}{p^3} + 12p + \frac{6}{p}$

(16) $25p^2 + \frac{1}{p^2} - 10;\ 125p^3 - \frac{1}{p^3} - 75p + \frac{15}{p}$

(19) $4p^2 + \frac{9}{p^2} + 12;\ 8p^3 + \frac{27}{p^3} + 36p + \frac{54}{p}$

(20) $9m^2 + \frac{16}{m^2} + 24,\ 27m^3 + \frac{64}{m^3} + 108m + \frac{144}{m}$

(22) $25p^2 + \frac{4}{p^2} - 20;\ 125p^3 - \frac{8}{p^3} - 150p + \frac{60}{p}$

(25) $\frac{x^2}{y^2} + \frac{y^2}{x^2} + 2;\ \frac{x^3}{y^3} + \frac{y^3}{x^3} + \frac{3x}{y} + \frac{3y}{x}$

(29) $\frac{m^2}{n^2} + \frac{n^2}{m^2} - 2;\ \frac{m^3}{n^3} - \frac{n^3}{m^3} - \frac{3m}{n} + \frac{3n}{m}$

7—8. Identities — Applications (b)

Let us learn to do sums making use of identities.

$(\sqrt{5})^2 = 5;\ (\sqrt{3})^2 = 3;\ (\sqrt{a})^2 = a.$

Let us now find the equivalent fractions of some irrational fractions.

1. $\frac{5}{\sqrt{3}+1} = \frac{5}{\sqrt{3}+1} \cdot \frac{\sqrt{3}-1}{\sqrt{3}-1}$

[Multiply the numerator and denominator by $\sqrt{3} - 1$]

$$= \frac{5(\sqrt{3}-1)}{(\sqrt{3})^2 - 1^2} \quad [\because (a+b)(a-b) = a^2 - b^2]$$

$$= \frac{5(\sqrt{3}-1)}{3-1} = \frac{5}{2}(\sqrt{3}-1)$$

The denominator has been converted into a rational number.

2. $\frac{4}{\sqrt{5}-1} = \frac{4}{\sqrt{5}-1} \cdot \frac{\sqrt{5}+1}{\sqrt{5}+1}$

$= \frac{4(\sqrt{5}+1)}{(\sqrt{5})^2-(1)^2} = \frac{4(\sqrt{5}+1)}{5-1} = \sqrt{5}+1$

3. $\frac{3}{\sqrt{5}+\sqrt{3}} = \frac{3}{\sqrt{5}+\sqrt{3}} \times \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}$

$= \frac{3(\sqrt{5}-\sqrt{3})}{(\sqrt{5})^2-(\sqrt{3})^2} = \frac{3(\sqrt{5}-\sqrt{3})}{5-3}$

$= \frac{3}{2}(\sqrt{5}-\sqrt{3})$

Exercise 7—8

Convert the denominators of the following fractions as rational numbers and rewrite the fractions with rational denominators.

(1) $\frac{1}{\sqrt{2}+1}$ (2) $\frac{1}{\sqrt{2}-1}$ (3) $\frac{3}{\sqrt{3}+2}$

(4) $\frac{5}{2-\sqrt{3}}$ (5) $\frac{1}{\sqrt{3}+\sqrt{2}}$ (6) $\frac{1}{\sqrt{3}-\sqrt{2}}$

(7) $\frac{4}{\sqrt{5}-\sqrt{3}}$ (8) $\frac{5}{\sqrt{3}-2}$

Answers

(2) $\sqrt{2}+1$

(6) $\sqrt{3}+\sqrt{2}$

8—1. Factorisation : $(a \pm b)^2$ form

We have learnt that $(a + b)^2 = a^2 + 2ab + b^2$ and $(a - b)^2 = a^2 - 2ab + b^2$

From this we can deduce that

$a^2 + 2ab + b^2 = (a + b)^2$ and $a^2 - 2ab + b^2 = (a - b)^2$; that is, $a^2 + 2ab + b^2 = (a + b)(a + b)$.

The factors of $a^2 + 2ab + b^2$ are $(a + b)$, $(a + b)$

In the same way the factors of $a^2 - 2ab + b^2$ are $(a - b)$, $(a - b)$.

Hence by making use of these identities, we can find the factors of some of the expressions which are perfect squares.

Examples :

1. $4a^2 + 20ab + 25b^2 = (2a)^2 + 2(2a)(5b) + (5b)^2$
$= (2a + 56)^2$

2. $9a^2 - 12ab + 4b^2 = (3a)^2 - 2(3a)(2b) + (2b)^2$
$= (3a - 2b)^2$

Exercise 8—1

I What term must be added to make each of the following expressions a perfect square?

(a) $25x^2 + 40xy$ (b) $49m^2 - 42mn$

(c) $16p^2 + 25q^2$ (d) $81a^2 + 72ab$

(e) $4x^2 - 28x$

II Find out whether the following are perfect squares. If they are perfect squares find their factors.

(1) $x^2 + 22x + 121$

(2) $x^2 - 8x + 16$

(3) $a^2 + 8x - 16$

(4) $a^2 + 4ab + 4b^2$

(5) $4a^4 + 4a^2b^2 + b^4$

(6) $64a^4 + 80a^2b^2 + 25b^4$

(7) $a^4 - 2a^2b^2 + 2b^2$

(8) $36x^2 - 12xy + b^2$

(9) $49m^4 - 56m^2n^2 + 16n^4$

(10) $9a^2 - 4ab + 64b^2$

Answers

I (a) $16y^2$ (b) $9x^2$ (c) $40pq$ (d) $16b^2$ (e) 49

II (1) yes $(x + 11)^2$ (2) Yes $(x - 4)^2$

(3) No (4) Yes $(a + 2b)^2$

(5) Yes $(2a^2 + b^2)^2$ (6) Yes $(8a^2 - 5b^2)^2$

(7) No (8) No

(9) Yes $(7m^2 - 4n^2)^2$ (10) No

8—2. Factorisation : $(a + b)(a - b)$ form (i)

In the VIII Standard you learnt that $(a + b)(a - b) = a^2 - b^2$.

From this you can deduce that $a^2 - b^2 = (a + b) \times (a - b)$.

Making use of this identity we easily find the factors of expressions which can be written as the difference of two perfect squares.

Examples :

1. $4p^2 - 9q^2 = (2p)^2 - (3q)^2 = (2p + 3q)(2p - 3q)$

2. $(2x + 3y)^2 - (x - 2y)^2$

$= [(2x + 3y) + (x - 2y)][(2x + 3y) - (x - 2y)]$

$= [2x + 3y + x - 2y]$

$[2x + 3y - x + 2y]$

$= [3a + y][x + 5y]$

3. $(a + b + c)^2 - (a + b)^2$

$= [(a + b + c) + (a + b)]\ [(a + b + c) - (a + b)]$

$= (a + b + c + a + b)\ (a + b + c - a - b)$

$= c\ (2a + 2b + c)$

Exercise 8—2

Factorise the following expressions :

I (1) $p^2 - q^2$ (2) $x^2 - y^2$

(3) $m^2 - n^2$ (4) $l^2 - m^2$

(5) $a^2 - b^2$ (6) $p^2 - 4q^2$

(7) $x^2 - 9y^2$ (8) $m^2 - 16n^2$

(9) $l^2 - 25m^2$ (10) $a^2 - 49b^2$

(11) $9p^2 - 4q^2$ (12) $16x^2 - 9y^2$

(13) $49m^2 - 16n^2$ (14) $16l^2 - 25m^2$

(15) $4a^2 - 49b^2$ (16) $9p^4 - 4q^4$

(17) $16x^4 - 9y^4$ (18) $49m^4 - 16n^4$

(19) $16l^6 - 25m^6$ (20) $4a^8 - 49b^8$

II (1) $(2x + 3y)^2 - (x + 2y)^2$

(2) $(3x - 2y)^2 - (x + y)^2$

(3) $(4a - 3b)^2 - (2a + 5b)^2$

(4) $(9p - 16q)^2 - (2p - 5q)^2$

(5) $(3m - 4n)^2 - (4m + 5n)^2$

(6) $(a + b + 2c)^2 - (b + c)^2$

(7) $(2a + 3b + 4c)^2 - (a + 4b - 3c)^2$

(8) $(3a - 2b + 4c)^2 - (2a - 3b - 4c)^2$

(9) $(x + y - z)^2 - (x - y + z)^2$

(10) $(4x - 5y - 7z)^2 - (2x - 3y + 8z)^2$

Answers

I (1) $(p + q)(p - q)$ (6) $(p + 2q)(p - 2q)$

(11) $(3p + 2q)(3p - 2q)$

(16) $(3p^2 + 2q^2)(3p^2 - 2q^2)$

II (1) $(3x + 5y)(x + y)$ (2) $(4x - y)(2x - 3y)$

(6) $(a + 2b + 3c)(a + c)$

8–3. Factorisation: (a + b) (a — b) form (ii)

Some of the expressions which at first sight are not in the form $a^2 - b^2$ can be rewritten in the above form and factorised.

Examples:

1. $4a^4 - b^4 = (2a^2)^2 + 2(2a^2)(b^2) + (b^2)^2 - 4a^2b^2$

$= (2a^2 + b^2)^2 - (2ab)^2$

$= (2a^2 + b^2 + 2ab)(2a^2 + b^2 - 2ab)$

$= (2a^2 + 2ab + b^2)(2a^2 - 2ab + b^2)$

2. $9x^4 - 33x^2y^2 + 16y^2$

$= (3x^2)^2 - 2(3x^2)(4y^2) + (4y^2)^2 - 9x^2y^2$

$[-33 = -24 - 9]$

$= (3x^2 - 4y^2)^2 - (3xy)^2$

$= (3x^2 - 4y^2 + 3xy)(3x^2 - 4y^2 - 3xy)$

$= (3x^2 + 3xy - 4y^2)(3x^2 - 3xy - 4y^2)$

3. $a^2 - b^2 - 4a - 6b - 5$

$= a^2 - 4a + 4 - b^2 - 6b - 9$

$= [a^2 - 2(a)(2) + 2^2]$

$- [b^2 + 2(b)(3) + 3^2]$

$= (a - 2)^2 - (b + 3)^2$

$= [(a - 2) + (b + 3)] [(a - 2) - (b + 3)]$

$= (a - 2 + b + 3) (a - 2 - b - 3)$

$= (a + b + 1) (a - b - 5)$

4. $(p^2 - q^2) (r^2 - s^2) + pqrs$

$= p^2r^2 - p^2s^2 - q^2r^2 + q^2s^2 + 4pqrs$ (on expansion)

$= (p^2r^2 + 2pqrs + q^2s^2) - (p^2s^2 - 2pqrs + q^2r^2)$

$= (pr + qs)^2 - (ps - qr)^2$

$= [(pr + qs) + (ps - qr)] [(pr + qs) - (ps - qr)]$

$= (pr + qs + ps - qr) (pr + qs - ps + qr)$

Exercise 8—3

Factorise :

(1) $x^2 + 9y^2 - 25z^2 - 6xy$

(2) $a^2 + b^2 - c^2 - d^2 + 2ab - 2cd$

(3) $a^2 + 8a + 4^2 - b^2$

(4) $9 - x^2 + 2xy - y^2$

(5) $y^2 - 2y - 17 + 4y^2$

(6) $4a^2 - 12a + 9 - b^2 - 6b - 9$

(7) $4x^4 - 81y^4$

(8) $64a^4 + 81b^4$

(9) $a^2b^2 + c^2d^2 - a^2d^2 - b^2c^2$

(10) $4p^4 + 16p^2q^2 + 25q^4$

(11) $25x^2 - 79x^2y^2 + 49y^4$

(12) $36m^4 - 16m^2n^2 + n^4$

Answers

(1) $(x - 3y + 5z)(x - 3y - 5z)$

(2) $(a + b + c + d)(a + b - c - d)$

(3) $(a + b + 4)(a - b + 4)$

(8) $(8a^2 + 12ab + 9b^2)(8a^2 - 12ab + 9b^2)$

(9) $(b + d)(b - d)(a + c)(a - c)$

(10) $(5x^2 + 3xy - 7y^2)(5x^2 - 3xy - 7y^2)$

8—4. Factorisation : $(x + a)(x + b)$ form

Remember that $(x + a)(x + b) = x^2 + x(a + b) + ab.$

$(x - a)(x - b) = x^2 - x(a + b) + ab$

Note :

1. $(x + 3)(x + 6) = x^2 + (3 + 6)x + 3 \times 6$
$= x^2 + 9x + 18$

2. $(a + 5)(a + 7) = a^2 + (5 + 7)a + 5 \times 7$
$= a^2 + 12a + 35$

3. $(x - 3)(x - 6) = x^2 - (3 + 6)a + 3 \times 6$
$= x^2 - 9x + 18$

4. $(a - 5)(a - 7) = a^2 - (5 + 7)a + 5 \times 7$
$= a^2 - 12a + 35$

The sum of the factors of the last term of the expression becomes the coefficient of the middle term. Hence to factorise an expression of the form $x^2 + px + q$,

(1) Write the last term as a product of two of its factors.

(2) Choose that pair of factors whose sum becomes equal to 'p'.

(3) Let that pair be a, b.

If 'p' is positive the factors are $(x + a), (x + b)$

If 'p' is negative the factors are $(x - a), (x - b)$

Examples :

(1) **Factorise :** $x^2 + 13x + 36$

The various pairs of factors of 36 are

(1, 36), (2, 18), (3, 12), (4, 9), (6, 6)

Their sums are 37, 20, 15, 13, 12.

The suitable pair is (4, 9). Since the coefficient 13 of the middle term is positive the factors are $(x + 4)$, $(x + 9)$

Hence $x^2 + 13x + 36 = (x + 4)(x + 9)$

(2) **Factorise :** $x^2 - 11x + 24$

The various pairs of factors of 24 are

(1, 24), (2, 12), (3, 8), (4, 6)

Their sums are 25, 14, 11, 10,

The suitable pair is (3, 8)

Since the coefficient -11 of the middle term is negative, the factors are $(x - 3)$, $(x - 8)$.

Hence $x^2 - 11x + 24 = (x - 3)(x - 8)$

Exercise 8—4

I The product and the sum of two numbers are given below. Find the suitable pair of numbers.

(1)	32, 12	(2)	12, 7	(3)	30, 11
(4)	42, 17	(5)	6, 5	(6)	12, 13
(7)	81, 30	(8)	108, 14	(9)	36, 15
(10)	48, 16	(11)	45, — 14	(12)	40, — 13
(13)	42, — 13	(14)	35, — 36	(15)	28, — 11
(16)	1, — 2	(17)	64, — 16	(18)	12, — 7
(19)	16, — 10	(20)	20, — 9.		

II Factorise :

(1) $a^2 + 4x + 3$	(2) $x^2 + 9x + 14$
(3) $a^2 + 6a + 8$	(4) $p^2 + 5p + 4$
(5) $m^2 + 19m + 90$	(6) $x^2 - 5x + 6$
(7) $x^2 - 7x + 10$	(8) $a^2 - 15a + 54$
(9) $p^2 - 12p + 20$	(10) $m^2 - 12m + 72$
(11) $x^2 + 6x + 5$	(12) $x^2 - 8x + 15$
(13) $a^2 + 15a + 26$	(14) $p^2 - 16p + 63$
(15) $m^2 + 2m + 1$	(16) $x^2 - 17x + 66$
(17) $x^2 + 13x + 40$	(18) $a^2 - 7a + 6$
(19) $p^2 - 24p + 44$	(20) $m^2 + 20m + 36$

Answers

I (1) 8, 4 (2) 4, 3 (4) 14, 3 (11) $-9, -5$ (12) $-8, -5$

II (1) $(x + 3)(x + 1)$ (2) $(x + 7)(x + 2)$

(3) $(a + 4)(a + 2)$ (7) $(x - 5)(x - 2)$

(8) $(a - 9)(a - 6)$

8—5. Factorise: (x + a) x — b) form

Remember : $(x + a)(x - b) = x^2 + x(a - b) - ab.$

$(x - a)(x + b) = x^2 - x(a - b) - ab.$

Note :

Expression	Expansion	Coefficient of last term	Coefficient of them middle term
$(x+11)(x-2)$	$x^2+9x-22$	$-22(11\times-2)$	$9(11-2)$
$(x+7)(x-3)$	$x^2+4x-21$	$-21(7\times-3)$	$4(7-3)$
$(x-11)(x+2)$	$x^2-9x-22$	$-22(-11\times2)$	$-9(-11+2)$
$(x-7)(x+3)$	$x^2-4x-21$	$-21(-7\times3)$	$-4(-7+3)$

From this, to find the factors of expressions of the form $x^2 + px - q$ we can arrive at the following steps.

(1) Find the pairs of factors of the coefficient of the last term 'q'

(2) Select that pair (a, b) whose difference is equal to the coefficient of the middle term $(a > b)$

(3) If p is positive the factors are $(x + a), (x - b)$

(4) If p is negative the factors are $(x - a), (x + b)$

Examples :

(1) Factorise : $x^2 + 15x - 100$

The pairs of factors of 100 are (1, 100), (2, 50), (4, 25), (5, 20), (10, 10)

The difference between the respective factors are

99, 48, 21, 15, 0

The suitable pair is (5, 20).

The coefficient 15 of the middle term is positive.

Hence the factors are $(x + 20), (x - 5)$

$x^2 + 15x - 10 = (x + 20)(x - 5)$

(2) Factorise: $x^2 - 8x - 48.$

The pairs of factors of 48 are (1, 48), (2, 24), (3,16), (4, 12) (6, 8)

The difference between the respective factors are 47, 22, 13, 8, 2.

The suitable pair is (4, 12)

The coefficient —8 of the middle term is negative.

Hence the factors are $(x + 4), (x - 12)$

$x^2 - 8x - 48 = (x + 4)(x - 12)$

Exercise 8—5

I The product and the difference of two numbers are given below. Find the suitable pair of numbers.

(1)	12, 1	(2)	56, 10	(3)	66, 5
(4)	72, 14	(5)	36, 5	(6)	64, 0
(7)	24, 2	(8)	30, 7	(9)	84, 5
(10)	48, 22	(11)	55, 54	(12)	12, 11
(13)	100, 0	(14)	55, 6	(15)	28, 3
(16)	40, 3	(17)	34, 15	(18)	108, 3
(19)	120, 19	(20)	135, 6.		

II Factorise:

(1)	$x^2 + 7x - 60$	(2)	$p^2 + 3p - 54$
(3)	$m^2 + 8m - 105$	(4)	$a^2 + 4a - 45$
(5)	$x^2 + 2x - 120$	(6)	$x^2 - 9x - 22$
(7)	$p^2 - 5p - 84$	(8)	$m^2 - 3m - 180$
(9)	$a^2 - 10a - 24$	(10)	$x^2 - 6x - 27$
(11)	$x^2 - 16$	(12)	$p^2 - 25$
(13)	$m^2 + m - 72$	(14)	$a^2 - 2a - 35$
(15)	$x^2 + 3x - 130$	(16)	$x^2 - 2x - 99$
(17)	$p^2 + p - 132$	(18)	$m^2 - 7m - 260$
(19)	$a^2 - 5a - 150$	(20)	$x^2 + 8x - 9$

Answers

I (1) 4, 3 (2) 14, 4 (13) 10, 10 (15) 7, 4

II (1) $(x + 12)(x - 5)$ (2) $(p + 9)(p - 6)$

(3) $(m + 15)(m - 7)$ (7) $(p - 12)(p + 7)$

(8) $(m - 15)(m + 12)$ (15) $(x + 3)(x - 10)$

(20) $(x + 9)(x - 1)$

8 6. Factorisation: $ax^2 + bx + c$ form (ac a positive number)

Remember: $a(b + c) = ab + ac$

$$(x + a)(y + b) = x(y + b) + a(y + b)$$

Observe the following carefully.

Expression	Expansion
$(2x + 5)\ (3x + 4)$	$6x^2 + 23x + 20$
$(4x + 3)\ (6x + 7)$	$24x^2 + 46x + 21$
$(2x - 5)\ (3x - 4)$	$6x^2 - 23x + 20$
$(4x - 3)\ (6x - 7)$	$24x^2 - 46x + 21$

Coefficient of the middle term	Coefficient of the product of the extremes
$23 = 8 + 15$	$6 \times 20 = 120 = 8 \times 15$
$46 = 28 + 18$	$24 \times 21 = 504 = 28 \times 18$
$-23 = -8 - 15$	$6 \times 20 = 120 = -8 \times -15$
$-46 = -28 - 18$	$24 \times 21 = 504 = -28 \times -18$

From the above we can understand that to factorise expressions of the form $ax^2 + bx + c$,

(1) Multiply the coefficients a, c of the extreme terms

(2) Find the pairs of the factors of a, c

(3) When ac is positive choose the pair whose sum is equal to 'b'

(4) Then factorise.

Examples :

(1) **Factorise:** $8x^2 + 28x + 20$.

The product of the coefficients of the extremes is $8 \times 20 = 160$

The pairs of factors are (1, 160), (2, 80), (4, 40), (5, 32) (8, 20), (10, 16).

The suitable pair for the coefficient 28 of the middle term is (8, 20).

$$8x^2 + 28x + 20 = 8x^2 + 8x + 20x + 20$$
$$= 8x(x + 1) + 20(x + 1)$$
$$= (8x + 20)(x + 1)$$
$$= 4(2x + 5)(x + 1)$$

(2) Factorise: $6a^2 - 19a + 15$

The product of the coefficient of the extremes is $6 \times 15 = 90$.

The pairs of factors are (1, 90), (2, 45), (3, 30), (5, 18), (6, 15), (9, 10).

The pair suitable for the coefficient —19 of the middle term is (9, 10).

$$6a^2 - 19a + 15 = 6a^2 - (9 + 10)a + 15$$
$$= 6a^2 - 9a - 10a + 15$$
$$= 3a(2a - 3) - 5(2a - 3)$$
$$= (2a - 3)(3a - 5)$$

Exercise 8—6

Factorise:

(1) $6x^2 + 5x + 1$ (2) $15x^2 + 8x + 1$

(3) $6x^2 + 7x + 2$ (4) $10p^2 + 11p + 3$

(5)	$6p^2 + 21p + 15$	(6)	$8p^2 + 25p + 17$
(7)	$6a^2 + 23p + 20$	(8)	$12m^2 + 43m + 35$
(9)	$6x^2 + 25x + 14$	(10)	$p^2 + 25p + 12$
(11)	$6x^2 - 5x + 1$	(12)	$15x^2 - 8x + 1$
(13)	$6x^2 - 7x + 2$	(14)	$10p^2 - 11p + 3$
(15)	$6p^2 - 21p + 15$	(16)	$8p^2 - 25p + 17$
(17)	$6a^2 - 23p + 20$	(18)	$12m^2 - 43m + 35$
(19)	$6x^2 - 25x + 14$	(20)	$12p^2 - 25p + 12$

Answers

(1)	$(3x + 1)(2x + 1)$	(2)	$(5x + 1)(3x + 1)$
(3)	$(3x + 2)(2x + 1)$	(11)	$(3x - 1)(2x - 1)$
(12)	$(5x - 1)(3x - 1)$	(15)	$3(2p - 5)(p - 1)$

8—7. Factorisation: $(ax^2 + bx + c)$ form (ac a negative number)

Example 1:

Note ; $(2x + 5)(3x - 4) = 6x^2 + 7x - 20$

The product of the coefficients of the extremes

$$= 6 \times 20 = 120 = 15 \times 8$$

The coefficient of the middle term $= 7 = 15 - 8$

If 'ac' is negative in the expression $ax^2 + bx + c$, choose that pair of factors whose difference is equal to 'b' the coefficient of the middle term.

$$6x^2 + 7x - 20 = 6x^2 + (15 - 8)x - 20$$
$$= 6x^2 + 15x - 8x - 20$$
$$= 3x(2x + 5) - 4(2x + 5)$$
$$= (2x + 5)(3x - 4)$$

Example 2

Factorise: $12x^2 - 13x - 35$

The product of the coefficients of the extremes

$= 12 \times -35 = -420.$

The pairs of factors of 420 are

(1, 420), (2, 210), (3, 140), (4, 105),

(5, 84), (6, 70), (7, 60), (10, 42),

(12, 35), (14, 30), (15, 28), (20, 21)

The pair whose difference is equal to 13 of the middleterm is (15, 28.)

$$\begin{aligned} 12x^2 - 13x - 35 &= 12x^2 - (28 - 15)\,x - 35 \\ &= 12x^2 - 28x + 15x - 35 \\ &= 4x\,(3x - 7) + 5\,(3x - 7) \\ &= (3x - 7)(4x + 5) \end{aligned}$$

Exercise 8—7

Factorise:

(1) $6x^2 + 5x - 11$ (2) $3x^2 + 5x - 8$

(3) $5x^2 + 4x - 9$ (4) $8x^2 + 11x - 19$

(5) $6p^- + p - 15$ (6) $10m^2 + 14m - 15$

(7) $12a^2 + 5a - 3$ (8) $15x^2 + 11x - 12$

(9) $14p^2 + 15p - 9$ (10) $9p^2 - 25$

(11) $6x^2 - 5x - 11$ (12) $3x^2 - 5x - 8$

(13) $5x^2 - 4x - 9$ (14) $8x^2 - 11x - 9$

(15) $6p^2 - p - 15$ (16) $10m^2 - 19m - 15$

(17) $12a^2 - 5a - 3$ (18) $15x^2 - 11x - 12$

(19) $14p^2 - 15p - 9$ (20) $8x^2 - 10x - 25$

Answers

(1) $(6x + 11)(x - 1)$ (2) $(3x + 8)(x - 1)$

(3) $(5x + 9)(x - 1)$ (4) $(8x + 19)(x - 1)$

(10) $(3p + 5)(3p - 5)$ (20) $(2x - 5)(4x + 5)$

9—1. Linear Equations — single variable — Revision

Recall:

$$a + 0 = 0 + a = a, \quad a + (-a) = (-a) + a = 0$$

$$a \times 1 = 1 \times a = a; \quad a \times \frac{1}{a} = \frac{1}{a} \times a = 1$$

Let us revise what we have studied about linear equations with a single variable in the VII and VIII standards.

(1) If $x + 5 = 6$

$$x + 5 + (-5) = 6 + (-5)$$
$$x + 0 = 1$$
$$x = 1$$

It can simply be solved as follows:

$x + 5 = 6$

$x = 6 - 5 = 1$

Solution set $\{ 1 \}$

(2) If $x - 3 = 9$

$$x - 3 + 3 = 9 + 3$$
$$x + 0 = 12$$
$$x = 12$$

This can be solved as follows:

$x - 3 = 9$

$x = 9 + 3 = 12$

Solution set $\{ 12 \}$

(3) If $8x = 24$

$$\frac{1}{8} \cdot 8x = \frac{1}{8} \cdot 24$$

$$1 \cdot x = 3$$

$$x = 3$$

or simply

$$8x = 24$$

$$x = \frac{24}{8} = 3$$

Solution { 3 }

(4) If $\frac{2}{3} x = 10$

$$\frac{3}{2} \cdot \frac{2}{3} x = \frac{3}{2} \cdot 10$$

$$1 x = 15$$

$$x = 15$$

or simply

$$\frac{2}{3} x = 10$$

$$x = 10 \times \frac{3}{2} = 15$$

Solution set { 15 }

(5) $2x + 3 = 13$

$$2x + 3 + (-3) = 13 + (-3)$$

$$2x + 0 = 10$$

$$2x = 10$$

$$\frac{1}{2} \cdot 2x = \frac{1}{2} \cdot 10$$

$$1 \cdot x = 5$$

$$x = 5$$

or simply

$2x + 3 = 13$

$2x = 13 - 3 = 10$

$x = \frac{10}{2} = 5$

Solution set $\{5\}$

(6) $\frac{5}{2}x - 4 = -19$

$\frac{5}{2}x - 4 + 4 = -19 + 4$

$\frac{5}{2}x + 0 = -15$

$\frac{5}{2}x = -15$

$\frac{2}{5} \cdot \frac{5}{2}x = \frac{2}{5} \times (-15)$

$1 \cdot x = -6$

$x = -6$

or simply

$\frac{5}{2}x - 4 = -19$

$\frac{5}{2}x = -19 + 4 = -15$

$x = -15 \times \frac{2}{5} = -6$

Solution set $\{-6\}$

Exercise 9—1 (Mental sums)

Find the solution sets.

1. **(a)** $x + 3 = 8$ **(b)** $a + 6 = 9$

(c) $x + 12 = 6$ **(d)** $x + 7 = 5$

(e) $x + 2\frac{1}{2} = 5$ (f) $x + 3 = -3$

(g) $a + 6 = -9$ (h) $x + 12 = -6$

(i) $x + 7 = -5$ (j) $x + 2\frac{1}{2} = -5$

2. (a) $x - 3 = 8$ (b) $a - 6 = 9$

(c) $x - 12 = 6$ (d) $x - 7 = 5$

(e) $x - 2\frac{1}{2} = 5$ (f) $x - 3 = -8$

(g) $a - 6 = -9$ (h) $x - 12 = -6$

(i) $x - 7 = -5$ (j) $x - 2\frac{1}{2} = -5$

3. (a) $3x = 12$ (b) $4x = 24$

(c) $2x = 7$ (d) $7x = -14$

(e) $2x = 6{\cdot}5$ (f) $3x = -12$

(g) $4x = -24$ (h) $-2x = 7$

(i) $-7x = -14$ (j) $2x = -6{\cdot}5$

4. (a) $\frac{2}{5}x = 10$ (b) $\frac{5}{2}x = 25$

(c) $\frac{4}{3}x = 12$ (d) $\frac{4}{7}x = 16$

(e) $\frac{3}{5}x = 4{\cdot}5$ (f) $\frac{5}{7}x = 12$

(g) $\frac{8}{9}x = 20$ (h) $\frac{3}{7}x = 16$

(i) $\frac{4}{9}x = 14$ (j) $\frac{9}{5}x = 40{\cdot}5$

5. (a) $3x + 4 = 10$ (b) $3x + 4 = -2$

(c) $2x + 5 = 15$ (d) $4x - 7 = 5$

(e) $2x - 6 = 4$ (f) $5x - 3 = -3$

(g) $4x + 9 = -3$ (h) $7x - 5 = 2$

(i) $2x + 5 = -9$ (j) $10x - 12 = 8$

6. (a) $\frac{2}{3}x + 5 = 7$ (b) $\frac{3}{5}x - 2 = 4$

(c) $\frac{7}{8}x + 5 = 26$ (d) $\frac{9}{5}x - 4 = 5$

(e) $\frac{3}{7}x - 6 = 3$ (f) $\frac{7}{3}x + 12 = 40$

(g) $\frac{5}{2}x - 6 = 19$ (h) $\frac{7}{4}x - 6 = 11{\cdot}5$

(i) $\frac{2}{5}x + 4 = 5$ (j) $\frac{4}{9}x - 2 = 4$

9—2. Linear equations — Single Variable — Applications

Example 1 :

The sum of two numbers is 117. Their difference is 27. Find the numbers.

Since the difference is 27, if one number is taken as x, the other will be x + 27.

$$\text{Their sum} = x + x + 27 = 117$$

$$2x + 27 = 117$$

$$2x = 117 - 27 = 90$$

$$x = 45$$

One number is 45. The other one is 45 + 27 = 72

Example 2 :

The ages of two persons are in the ratio 3 : 4. 20 years ago their ages were in the ratio 1 : 2. What is the present age of each ?

If the age of one is 3x years, then the age of the second will be 4x years.

20 ago before their ages were $3x - 20$, $4x - 20$ respectively.

$$\therefore \quad (3x - 20) : (4x - 20) = 1 : 2$$

$$\therefore \quad 2(3x - 20) = 1(4x - 20)$$

$$6x - 40 = 4x - 20$$

$$6x - 4x = -20 + 40$$

$$2x = 20$$

$$x = 10$$

Their present ages are 30, 40.

Play with numbers :

Choose a number; multiply it by 5 and then add 3. Multiply the result by 8 and then subtract 24. Divide the result by 4 and find out the relationship between the chosen number and the resultant got.

If the chosen number is x then on multiplying it by 5 and adding 3 the resultant $= 5x + 3$.

The product of $(5x + 3)$ and $8 = 40x + 24$.

Subtracting 24 we get $40x + 24 - 24 = 40x$.

Dividing by 4 we get $\frac{40x}{4} = 10x$.

The resultant is 10 times the chosen number. Hence by deleting the zero from the resultant we get the chosen number.

Exercise 9—2

I. Frame equations, given the sum and difference of two numbers as follows :

	Sum	Difference
(a)	80	10
(b)	92	28
(c)	35	
(d)	48	16
(e)	75	25

2. The ages of two persons are in the ratio 2 : 3. After 10 years their ages will be in theratio 3 : 4. Find their ages.

3. The cost of two machines are in the ratio 4 : 5. If the cost of each machine decreases by Rs. 500 then they will be in the ratio 3 : 4. What are their costs at the beginning?

4. Two numbers are in the ratio 5 : 4. If 5 is added to the first number and 2 to the second then they will be in the ratio 4 : 3. Find the numbers.

5. The number of students in two classes are in the ratio 8 : 7. If 3 students from the first class are sent to the second, then the number of students in the two classes become equal. Find the actual number of students in the first class at the beginning.

6. The distance travelled by two persons are in the ratio 4 : 3. If the first one travels 60 km more and the second 120 km more then the ratio will be 7 : 9. Find the distance travelled by them.

Answers

1. (a) $x + y = 80$, $x - y = 10$ (b) $x + y = 92$, $x - y = 28$

(c) $x + y = 35$, $x - y = 5$ (d) $x + y = 48$, $x - y = 16$

(e) $x + y = 75$, $x - y = 25$

2. 20, 30 3. Rs. 2000, Rs. 2500 4. 35, 28
5. 48 6. 80 km; 60 km.

9—3. Linear Equations (Two variables)

Remember:

By multiplying or dividing both sides of an equation by the same number, we get another equivalent equation.

$2x + 3y = 5$, $4x + 6y = 10$, $6x + 9y = 15$, $14x + 21y = 35$ are all equivalent equations.

In the VIII standard we learnt to solve linear equations in two variables. Let us revise the same.

Example 1 : $2x + 3y = 1$ ... (1)

$3x - 4y = 10$ ... (2)

The l. c. m. of the coefficients of y namely 3, 4 is 12.

(1) × 4 $8x + 12y = 4$ ... (3)

(2) × 3 $9x - 12y = 30$ ... (4)

(3) + (4) $(8x + 9x) + (12y - 12y) = 4 + 30$

$17x + 0 = 34$

$x = 34 \div 17 = 2.$

Substituting the value of x in (1)

$4 + 3y = 1$

$3y = 1 - 4 = -3$

$y = -3 \div 3 = -1$

$x = 2, y = -1$ or the solution set is $\{ (2, -1) \}$

Example 2 :

$3x + 4y = 6$ ... (1)

$5x + 6y = 8$ ... (2

The l. c. m. of 4, 6 is 12.

(1) × 3 $\quad 9x + 12y = 18 \quad$ ... (3)

(2) × — 2 $\quad -10x - 12y = -16 \quad$ ... (4)

(3) + (4) $\quad (9x - 10x) + (12y - 12y) = (18 - 16)$

$$-x + 0 = 2$$

$$x = -2$$

Substituting the value of x in (1)

$$-6 + 4y = 6$$

$$4y = 6 + 6 = 12$$

$$y = 12 \div 4 = 3$$

Solution set { (— 2, 3) }

Example 3 :

$4x - 3y = 14{\cdot}5 \quad$ ... (1)

$2x - 5y = 12{\cdot}5 \quad$ ... (2)

The l. c. m. of 3, 5 is 15.

(1) × 5 $\quad 20x - 15y = 72{\cdot}5 \quad$ ... (3)

(2) × — 3 $\quad -6x + 15y = -37{\cdot}5 \quad$ ... (4)

(3) + (4) $\quad (20x - 6x) + (-15y + 15y) = 72{\cdot}5 - 37{\cdot}5$

$$14x = 35$$

$$x = 35 \div 14 = 2{\cdot}5$$

Substituting the value of x in (1)

$$10 - 3y = 14{\cdot}5$$

$$-3y = 14{\cdot}5 - 10 = 4{\cdot}5$$

$$y = 4{\cdot}5 \div -3$$

$$= -1.5$$

Solution set = { (2·5, — 1·5) }

Exercise 9—3

I For each one of the following equations given on the left, find out the equivalent equation from the right.

(a) $2x + 5y = 8$

$5x + 2y = 8$
$6x + 15y = 24$
$4x + 10y = 8$
$2x - 5y = 8$

(b) $3x - 4y = 6$

$9x - 12y = 2$
$3x + 4y = 6$
$15x - 20y = 30$
$4x - 3y = 6$

(c) $5x + 3y = 12$

$3x + 5y = 12$
$15x + 9y = 12$
$15x + 13y = 36$
$20x + 12y = 48$

(d) $1{\cdot}5x + 2{\cdot}25y = 4$

$6x + 9y = 16$
$3x + 4{\cdot}5y = 4$
$4{\cdot}5x + 6{\cdot}5y = 12$
$6x + 9y = 4$

II Find the solution set for the following equations :

1. (i) $2x + 3y = 5$, $4x - 3y = -1$
   (ii) $4x + 5y = 13$, $3x - 5y = 1$
   (iii) $5x + 2y = 9$, $3x - 2y = -1$
   (iv) $5x + 2y = 6$, $3x - 2y = 10$

2. (i) $2x + 3y = 5$, $3x - 4y = -1$
   (ii) $3x + 5y = 11$, $2x - 3y = 1$
   (iii) $2x + 5y = 3$, $3x - 2y = -5$
   (iv) $3x + 4y = -11$, $2x - 3y = 4$

3. (i) $2x + 3y = 5$, $3x + 4y = 7$
   (ii) $4x + 5y = 13$, $2x + 3y = 7$
   (iii) $2x + 5y = 3$, $3x + 2y = -1$
   (iv) $3x + 4y = -11$, $2x + 3y = -8$

4. (i) $2x - 3y = -1$
$3x - 4y = -1$
(ii) $4x - 5y = 3$
$2x - 3y = 1$
(iii) $2x - 5y = -7$
$3x - 2y = -5$
(iv) $3x - 4y = 5$
$2x - 3y = 4$

Answers

II 1. (i) $\{(1, 1)\}$ 2. (iii) $\{(-1, 1)\}$
3. (i) $\{(1, 1)\}$ 4. (iii) $\{(-1, 1)\}$

9—4. Linear equations — Two variables — Applications

We learnt how to form linear equations in two variables.

1. (i) The cost of two 'Writer' pens and three 'Doctor' pens is Rs. 14. If the cost of one 'Writer' pen is Rs. x and that of the 'Doctor' pen is Rs. y, then

The cost of two 'Writer' pens is = Rs. 2x

The cost of three 'Doctor' pens is = Rs. 3y

The total cost of two 'Writer' pens and three 'Doctor' pens is } $= 2x + 3y = 14$

(ii) The cost of three 'Doctor' pens exceeds the cost of two 'Writer' pens by Rs. 4.

The cost of three 'Doctor' pens
= The cost of two 'Writer' pens + Rs. 4

$$3y = 2x + 4$$

$$-2x + 3y = 4$$

(iii) The cost of two 'Writer' pens is Rs. 4 less than that of three 'Doctor' pens.

The cost of two 'Writer' pens
= The cost of three 'Doctor' pens — Rs. 4

$$2x = 3y - 4$$

$$2x - 3y = -4$$

2. The sum of two numbers is 100. Their difference is 40.

If the numbers are x, y $(x > y)$, then

$$\text{their sum} = x + y$$

$$\text{difference} = x - y$$

Hence $x + y = 100$; $x - y = 40$

3. (i) The ages of two persons are in the ratio 2 : 3

If their ages are x, y, then $x : y = 2 : 3$

$$3x = 2y$$

$$3x - 2y = 0$$

(ii) Fifteen years back their ages were in the ratio 1 : 2

The age of the first one before 15 years $= x - 15$

The age of the second one before 15 years $= y - 15$

From the given facts $(x - 15) : (y - 15) = 1 : 2$

$$2(x - 15) = 1(y - 15)$$

$$2x - 30 = y - 15$$

$$2x - y = -15 + 30 = 15$$

4. The sum of the digits of a two digit number is 9.

The value of the number increases by 45 if the digits are interchanged.

Let the number be xy. That is, the units digit is y and the tens digit is x.

Sum of the digits $x + y = 9$

Value of the number $= x$ tens $+ y$ units $= 10x + y$

If the digits are interchanged the new number is yx. x becomes the units digit and y the tens digit.

Value of the number $= y$ tens $+ x$ units $= 10y + x$

Value of the new number = Value of the old number + 45

$$10y + x = 10x + y + 45$$

$$x + 10y - y - 10x = 45$$

$$-9x + 9y = 45$$

ie $$-x + y = 5 \quad \text{(Dividing by 9)}$$

5. A table is sold at a profit of 10% and a chair at a profit of 5%. The total profit is Rs. 50.

Let the cost of the table be Rs. x and that of the chair Rs. y.

Profit on the table $= \frac{10}{100} \times x = \frac{10x}{100}$

Profit on the chair $= \frac{5}{100} \times y = \frac{5y}{100}$

Total profit $= \frac{10x}{100} + \frac{5y}{100}$

Hence $\frac{10x}{100} + \frac{5y}{100} = 50$

Multiplying by 100, $10x + 5y = 5000$

Dividing by 5, $2x + y = 1000$

6. A table is sold at a profit 10% and a chair at a loss of 5%. The net profit is Rs. 10.

Profit on the table $= \frac{10}{100} \times x = \frac{10x}{100}$

Loss on the chair $= \frac{5}{100} \times y = \frac{5y}{100}$

Net profit = Profit on the table—Loss on the chair

$$= \frac{10x}{100} - \frac{5y}{100}$$

Hence $\frac{10x}{100} - \frac{5y}{100} = 10$

or $10x - 5y = 1000$

$2x - y = 200$

7. The length of a rectangular room is increased by 4 metres and the breadth reduced by 2 metres. Its base area increases by 8 m^2.

If the length is 'l' metres and the breadth 'b' metres then its area $= l\,b\ m^2$

The new length after an increare of 4 m $= l + 4$

The new breadth after a reduction of 2 m $= b - 2$

The base area of the new room $= (l + 4)(b - 2)$

$= lb - 2l + 4b - 8$

New area = old area + 8

$lb - 2l + 4b - 8 = lb + 8$

$lb - 2l + 4b - lb = 8 + 8$

$-2l + 4b = 16$

$-l + 2b = 8$

Exercise 9—4

1. Frame equations from the following data and solve them.

	No. of first items	No. of second items	Total cost Rs.
(a)	5	4	23.00
	2	5	16.00
(b)	4	7	45.00
	3	2	17.50
(c)	2	3	9.00
	5	2	11.50

2. (a) The price of three English books exceeds the price of two mathematics books by Re. 1. The price of five English books is Re. 1 less than that of four mathematics books.

(b) The price of 8 pens is Rs. 5 more than the price of 5 instrument boxes. The price of 2 pens is Rs. 2 more than that of 1 instrument box.

(c) The cost of 4 Philips tube lights is Rs. 60 less than that of 6 Bajaj tube lights. The cost of 3 Philips tube lights is Rs. 55 more than that of 2 Bajaj tube lights.

3. (a) The sum of two numbers is 120. Their difference is 20.

(b) The sum of two numbers is 175. Their difference is 25.

(c) The sum of two numbers is 110. Their difference is 0.

4. (a) The ages of two persons are in the ratio 3 : 5. Before ten years their ages were in the ratio 1 : 2.

(b) The amount possessed by two persons are in the ratio 7 : 4. If each of them spends Rs. 2,000 the ratio will be 16 : 7.

(c) Ten years ago the ages of a father and his son were in the ratio 1 : 6. After ten years they will be in the ratio 1 : 2..

5. (a) The sum of the digits of a two digit number is 8. If the digits are interchanged its value decreases by 36.

(b) The sum of the digits of a two digit number is 15. If the digits are interchanged its value increases by 9.

(c) The difference of the digits of a two digit number is 3. When the digits are interchanged their value increases by 27.

6.

	Gain/Loss on the first item	Gain/Loss on the second item	Total Gain/ Loss
(a)	15% Gain	10% Gain	Rs. 250 Gain
	10% Gain	15% Gain	Rs. 250 Gain

(b) 8% Gain	5% Loss	Rs. 50 Gain
10% Gain	5% Gain	Rs. 130 Gain
(c) 12% Gain	10% Loss	Rs. 2 Loss
5% Loss	8% Gain	Rs. 8.50 Gain

7.

Change in the length of a rectangular room	Change in breadth	Change in area
(a) decreases by 3 m.	increases by 2 m.	increases by 1 m^2.
decreases by 2 m.	increases by 3 m.	increases by 12 m^2.
(b) increases by 10 m.	decreases by 5 m.	no change
decreases by 10 m.	increases by 15 m.	increases by 400 m^2.
(c) increases by 10 m.	decreases by 10 m.	decreases by 200 m^2.
decreases by 10 m.	increases by 10 m.	no change

Answers

2. (a) Rs. 3, Rs. 4 (b) Rs. $2\frac{1}{2}$, Rs. 3

   (c) Rs. 45, Rs. 40

3. (a) 70, 50 (b) 100, 75 (c) 55, 55

4. (a) 30, 50 (b) Rs. 8,400, Rs. 4.800 (c) 15, 40

5. (a) 62 (b) 78 (c) 36

6. (a) Rs. 1,000, Rs. 1,000

7. (a) 8 m; 3 m.

MATHEMATICS CLUB — ACTIVITY 3

To express a number as a difference of two squares:

$48 = 1 \times 48 = 2 \times 24 = 3 \times 16 = 4 \times 12 = 6 \times 8$

$$48 = \left(\frac{48 + 1}{2}\right)^2 - \left(\frac{48 - 1}{2}\right)^2$$

$$= \left(\frac{24 + 2}{2}\right)^2 - \left(\frac{24 - 2}{2}\right)^2$$

$$= \left(\frac{16 + 3}{2}\right)^2 - \left(\frac{16 - 3}{2}\right)^2$$

$$= \left(\frac{12 + 4}{2}\right)^2 - \left(\frac{12 - 4}{2}\right)^2$$

$$= \left(\frac{8 + 6}{2}\right)^2 - \left(\frac{8 - 6}{2}\right)^2$$

$$= 24{\cdot}5^2 - 23{\cdot}5^2 = 13^2 - 11^2 = 9{\cdot}5^2 - 6{\cdot}5^2 = 8^2 - 4^2$$
$$= 7^2 - 1^2$$

Of these $(13^2 - 11^2)$, $(8^2 - 4^2)$, $(7^2 - 1^2)$ are from the set of integers.

Similarly any number can be expresed as a difference of two squares.

The number can be written in the form

$(a^2 + b^2)\ (c^2 + d^2) = (ac + bd)^2 + (ad - bc)^2$

This can he verified.

Take two numbers each of which is the sum of two squares. The product of such numbers can be expressed as the sum of two squares.

$13 = 2^2 + 3^2, \quad 41 = 4^2 + 5^2$

$a = 2, \quad b = 3, \quad c = 4, \quad d = 5$

$ac + bd = 8 + 15 = 23$

$ad - bc = 10 - 12 = -2$

$533 = 13 \times 41 = 23^2 + 2^2 = (2^2 + 3^2)\ (4^2 + 5^2)$

Try if you can cxpress 612, 725 as the sum of two squares.

Euler, the Swiss mathematician, found the method of expressing the product of two numbers which could each be expressed as the sum of 4 numbers, as the sum of 4 squares. Liouville tried to express a number as the sum of the fourth powers of some numbers.

MATHEMATICS CLUB — ACTIVITY 4

Perfect Numbers

If the sum of the divisors excluding the number itself is equal to the number, that number is called a perfect number.

The divisors of 28 : 1, 2, 4, 7, 14

Sum of the divisors : $1 + 2 + 4 + 7 + 14 = 28$

28 is a perfect number.

Find the smallest perfect number.

Verify if $2^{n-1}(2^n - 1)$ for $n = 2, 3, 5, 7$ gives perfect numbers.

n	$2^n - 1$	2^{n-1}	$2^{n-1}(2^n - 1)$	In base 2
2	3	2	6	110
3	7	4	28	11100
5	31	16	496	111110000
7	...	...	...	...

If it is given that $n = 13$, $n = 17$ also give perfect numbers can you write down these numbers in base two ?

Aren't you surprised to know that this formula was discoverd by Euclid some 2300 years back ?

4. MENSURATION

1—1. Pythagoras Theorem

In a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the sides containing the right angle. In Fig. 4-1,

$$AC^2 = AB^2 + BC^2$$

1—2. Right angled triangle

You have studied that the sides opposite to the equal angles are equal.

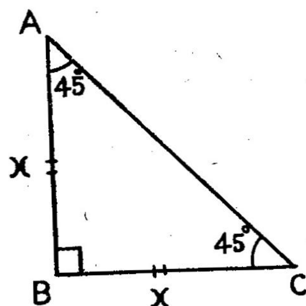


Fig. 4-1.

ABC is an isosceles triangle. Name the equal sides.

$\angle ABC$ is a right angle.

$$AB = BC$$

$$\therefore \; m\angle BAC = m\angle ACB = 45^\circ$$

By Pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$
$$= x^2 + x^2 = 2x^2$$
$$AC = \sqrt{2x^2}$$
$$= \sqrt{2}\,x$$

$$\therefore \; AB : BC : AC = x : x : \sqrt{2}\,x$$
$$= 1 : 1 : \sqrt{2}$$

If an angle is 45° in a right angled triangle, then the ratio of the sides is $1 : 1 : \sqrt{2}$ or, in an isosceles right triangle, the sides are in the ratio $1 : 1 : \sqrt{2}$.

Equilateral triangle

ABC is an equilateral triangle. AD is drawn perpendicular to BC.

$\therefore$ $\angle ADB$ is a right angle.

$\therefore$ $\triangle ADB$ is a right angled triangle and one angle is 30°. Why ?

What are the measures of the other angles?

AB = 2a, BD = a.

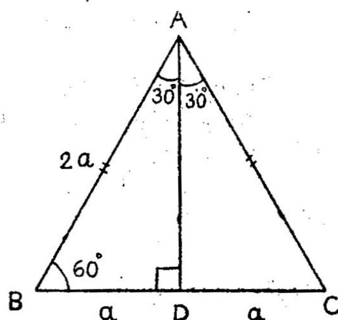


Fig. 4-2.

$$AD^2 = AB^2 - BD^2 = (2a)^2 - a^2$$
$$= 4a^2 - a^2 = 3a^2$$
$$\therefore \quad AD = \sqrt{3a^2} = \sqrt{3}\,a$$
$$BD : AD : AB = a : \sqrt{3}\,a : 2a$$
$$= 1 : \sqrt{3} : 2$$

If one angle of a right angled triangle is 30°, then the ratio of the sides will be $1 : \sqrt{3} : 2$.

The side oppositc to 90° is 2

The side opposite to 30° is 1

The side opposite to 60° is $\sqrt{3}$

What can be understood from this?

Exercise 1—2

1. The measures of angles of a triangle are 30°, 60°, 90°. Calculate the following :

(a) The side opposite to 30° is 5 cm. Find the measures of the other sides.

(b) The side opposite to 60° is 6 cm. Calculate the measures of the other sides,

(c) The length of the side opposite to 90° is 10 cm. Calculate the lengths of the other sides.

2. Compute the altitudes of the following equilateral triangles of sides

(a) 4 cm (b) 10 cm (c) 6·4 cm (d) 120 m

3. The angle of elevation of the top of a tower from a place 30 m away from it is 60°. Compute its height.

4. Given the altitude of an equilateral triangle, find its side correct to one decimal place. ($\sqrt{3} \simeq 1{\cdot}732$)

(a) 8 cm (b) 10 cm (c) 40 m (d) 15 m

Answers

1. (a) 10 cm, 8·66 cm (b) 3·464 cm, 6·928 cm
(c) 5 cm, 8·66 cm.

2. (a) $2\sqrt{3}$ cm (b) $5\sqrt{3}$ cm (c) 5·5424 cm
(d) $60\sqrt{3}$ cm

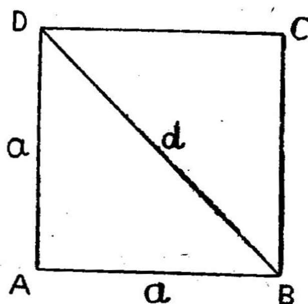
3. $30\sqrt{3}$ m

4. (a) 9·2 cm (b) 11·5 cm (c) 46·2 cm (d) 17·3 m

2—1. Area of a square in terms of its diagonals

ABCD is a square.

The length of its side is 'a'. What is its area ?



The diagonal of the square is d. How to find the area of a square in terms of its diagonal ?

$$a = \frac{d}{\sqrt{2}} \text{ (How ?)}$$

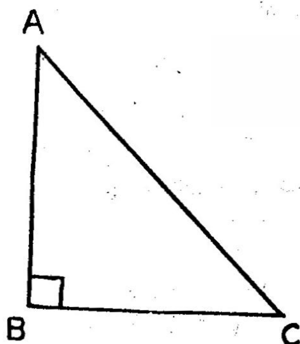
$d^2 = 2a^2$ (by Pythagoras Theorem)

Fig. 4-3.

$$\text{Area of the square} = a^2 = \frac{d^2}{2}$$

$$\text{The side of the square } a = \sqrt{\frac{d^2}{2}} = \frac{d}{\sqrt{2}}$$

2—2. Area of a right angled triangle



ABC is a right triangle.

Area $= \frac{1}{2} \times \text{base} \times \text{altitude}$

$= \frac{1}{2} \times BC \times AB$

Fig. 4-4.

Area of a right triangle

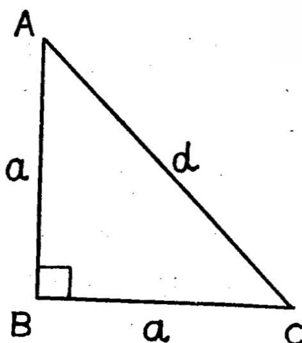
= Half the product of the sides containing the right angle.

2—3. Area of a right angled isosceles triangle

ABC is a right angled isosceles triangle.

The side is 'a' units; the hypotenuse is 'd' units.

What is its area?



Area of the triangle

$= \frac{1}{2} \times \text{base} \times \text{altitude}$

$= \frac{1}{2} \times a \times a$

$= \frac{a^2}{2}$

If $a = \frac{d}{\sqrt{2}}$

Fig. 4-5.

then area of the right angled isosceles triangle

$$= \frac{1}{2} \times \frac{d}{\sqrt{2}} \times \frac{d}{\sqrt{2}} = \frac{d^2}{4}$$

2—4. Area of an equilateral triangle

We know that if all the three sides of a triangle are equal, then that triangle is called an equilateral triangle.

What can you say about the angles when the sides are equal?

Construc an equilateral triangle and measure the angles.

AD is drawn perpendicular to BC.

D is the mid poin of BC.

$\therefore$ BD = DC

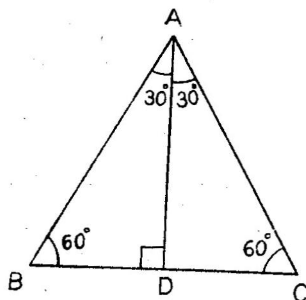


Fig. 4-6

AD bisects the angle A.

$\therefore \angle BAD = 30°$ and $\angle CAD = 30°$

ABD is a right angled triangle.

AB is the hypotenuse.

$$AD = \frac{\sqrt{3}}{2}a \quad \text{(a, side of the triangle)}$$

$$\text{Area of the triangle ABC} = \frac{1}{2} \times \text{base} \times \text{altitude}$$

$$= \frac{1}{2} \times a \times \frac{\sqrt{3}}{2}a$$

$$= \frac{\sqrt{3}}{4}a^2 \text{ or } \frac{a^2\sqrt{3}}{4}$$

$\therefore$ Area of an equilateral triangle is $\frac{a^2\sqrt{3}}{4}$

Exercise 2—4

1. Find the area of the following equilateral triangles with sides

(a) 6 cm (b) 10 cm (c) 60 m (d) 4·8 cm.

2. The length of the side of an equilateral field is 176 m. Find its area.

3. Find the side of an equilateral triangle whose area is $16 \times \sqrt{3}$ cm².

4. Find the area of an equilateral triangle whose altitude is $\sqrt{3} \times 7$ cm.

5. The sides containing the right angle of an isosceles triangle are given below. Find the area of the triangle.

(a) 5 cm (b) 7·2 cm (c) 10 m (d) 8·6 cm.

6. Find the length of the sides containing the right angle of a right angled isosceles triangle whose area is

(a) 18 cm² (b) 32 cm² (c) 24·5 cm² (d) 40·5 m².

Answers

1. (a) $9\sqrt{3}$ cm² (b) $25\sqrt{3}$ cm² (c) $900\sqrt{3}$ m²

(d) $5{\cdot}76\sqrt{3}$ cm²

2. $7744\sqrt{3}$ m² 3. 8 cm 4. $49\sqrt{3}$ cm²

5. (a) 12·5 cm² (b) 25·92 cm² (c) 50 m² (d) 36·98 cm²

6. (a) 6 cm (b) 8 cm (c) 7 cm (d) 9 cm.

2—5. To find the area of a regular hexagon

Construct 6 equilateral triangles of side 3 cm and paste them as shown in Fig. 4-7.

The resultant figure is a regular hexagon.

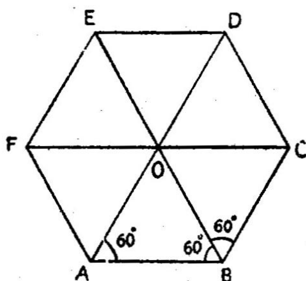


Fig. 4-7.

The measure of the side of the regular hexagon is 3 cm; one of its angles is 120°. Why?

We can conclude that the area of a regular hexagon is equal to the sum of the areas of the six equilateral triangles.

We know that the area of an equilateral triangle of side 'a' is $\frac{\sqrt{3}}{4}a^2$

From this we can infer that the area of a regular hexagon is $\frac{6\sqrt{3}\,a^2}{4}$

The area of a regular hexagon $= \frac{6\sqrt{3}}{4}a^2$ square units.

Exercise 2—5

1. What is the measure of an internal angle of a regular hexagon?

2. The perimeter of a regular hexagon is 7·2 m. Find its area.

3. Find the area of the regular hexagon of side

(a) 2 cm (b) 1 m (c) 10 m (d) 5 cm.

4. Find the area of the greatest regular hexagon which can be inscribed in a circle of radius 5·6 cm.

Answers

1. 120° 2 $2{\cdot}16\sqrt{3}\ m^2$ 3. (a) $6\sqrt{3}\ cm^2$

(b) $\frac{3\sqrt{3}}{2}\ cm^2$ (c) $150\sqrt{3}\ m^2$ (d) $\frac{75\sqrt{3}}{2}\ cm^2$

4. $47{\cdot}04\sqrt{3}\ cm^2$

3. To find the area of a triangle when three sides are given

a, b, c are the sides of the triangle ABC.

Area of any triangle $= \frac{1}{2} \times$ base $\times$ altitude

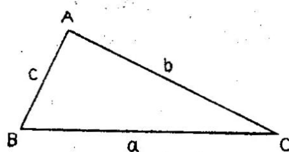


Fig. 4.8.

The area of a triangle whose sides are given is

$$\sqrt{s(s-a)\ (s-b)\ (s-c)}$$

where, 's' is the semi perimeter which is equal to $\frac{a+b+c}{2}$.

The formula is known as 'Hero's formula' after the Greek Mathematician who discovered this. Some say it is Heron, not Hero.

Example :

The sides of a triangle are of lengths 4 cm, 5 cm and 7 cm. Find its area.

$$s = \frac{a+b+c}{2} = \frac{4+5+7}{2} = \frac{16}{2} = 8\ cm.$$

Area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{8(8-4)(8-5)(8-7)}$

$= \sqrt{8 \times 4 \times 3 \times 1}$

$= \sqrt{96}$

$= 9{\cdot}8 \text{ cm}^2$

(correct to one decimal)

	9·8	
9	96	
	81	
188	1500	
	1504	
	0	

Exercise 3

1. Find the area of the following triangles of sides

   (a) 12 cm, 35 cm, 37 cm. (b) 36 m, 61 m, 65 m.
   (c) 5·4 cm, 9 cm, 7·2 cm. (d) 17 m, 25 m, 26 m.

2. The sides of a triangular plane are 75 m, 65 m and 20 m. Find the cost of levelling it at the rate of 10 ps per square metre.

3. The sides of a triangular field are in the ratio 8 : 15 : 17. Find its perimeter if its area is 135 ares.

Answers

1. (a) 210 cm² 2. Rs. 60 3. 600 m

4—1. Length of an arc of a sector of a circle

Look at the figures. We know that the angle subtended at the centre O in Fig. 4-9 is 360° and the circumference of the circle is $2\pi r$.

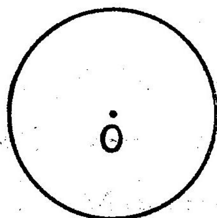


Fig. 4-9.

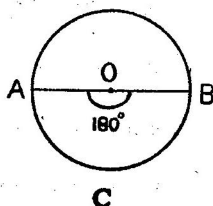


Fig 4-10

The circumference of the full circle $= 2\pi r$

The circumference of the semi-circle $= \frac{1}{2} \times 2\pi r$

$= \pi r$

This is the length of arc ACB.

The angle at O is 180° and this is half of the angle at the centre of a circle, i.e. half of 360°.

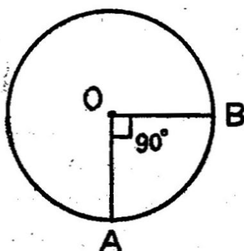


Fig. 4-11.

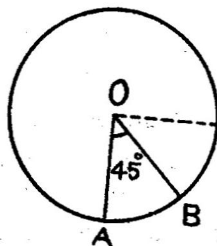


Fig. 4-12.

In Fig. 4-11, OAB is the sector of the circle and the angle at the centre is 90°.

This is $\frac{1}{4}$ of the angle at the centre of the circle i.e. 360°.

Compare the length of AB with the circumference of the circle.

We can see that it is $\frac{1}{4}$ of the circumference of the circle.

$\therefore$ The length of arc AB $= \frac{1}{4} \times 2\pi r$

In Fig. 4-12, OAB is the sector and the angle at its centre is 45°. This is $\frac{1}{8}$ of the angle at the centre of the circle and we can also see that the length of the arc AB is $\frac{1}{8}$ of the circumference of the circle.

i.e. The length of the arc AB $= \frac{1}{8} \times 2\pi r$

Angle at the centre	Length of arc
360°	$2\pi r$ (Circumference of a circle)
180°	$\frac{1}{2} \times 2\pi r$ $\left(\frac{180}{360} \times 2\pi r\right)$
90°	$\frac{1}{4} \times 2\pi r$ $\left(\frac{90}{360} \times 2\pi r\right)$
45°	$\frac{1}{8} \times 2\pi r$ $\left(\frac{45}{360} \times 2\pi r\right)$

From this we see that the ratio of the angle subtended at the centre of the sector to 360° is equal to the ratio of the length of arc to the circumference of the circle.

If D is the angle at the centre and l is the length of arc, then

$$l = \frac{D}{360} \times 2\pi r$$

Example 1 :

Given that the angle at the centre of a sector is 210° and the radius is 9 cm, calculate the length of its arc.

$$l = \frac{D}{360} \times 2\pi r$$

$$= \frac{210}{360} \times 2 \times \frac{22}{7} \times 9$$

$$= 33 \text{ cm.}$$

Example 2 :

The radius and length of arc of a sector are 10·5 cm and 22 cm respectively. Find the angle at the centre.

$$l = \frac{D}{360} \times 2\pi r$$

$$D = \frac{l \times 360}{2\pi r}$$

$$= \frac{22 \times 360 \times 7}{2 \times 22 \times 10{\cdot}5} = 120$$

∴ The angle at the centre of the sector is 120°.

Exercise 4—1

1. The following are the angles of a sector. Find the length of arc as a fraction of the circumference of the circle.

(a) 20° (b) 24° (c) 72° (d) 144°

2. The radius and the angle of a sector are given below. Find the length of arc.

radius :	(a) 7 cm	(b) 10·5 cm	(c) 21 cm	(d) 15 cm
angle :	270°	135°	60°	84°

3. The length of arc and the angle of a sector are given below. Find its radius.

length of arc:	(a) 11 cm	(b) 33 cm	(c) 52·8 cm
angle:	63°	135°	216°

4. The length of arc and radius of a sector are given below. Find its angle.

length of arc:	(a) 55 cm	(b) 99 cm	(c) 88 cm
radius	21 cm	55 cm	64 cm

Answers

1. (a) $\frac{1}{18}$ (b) $\frac{1}{12}$ (c) $\frac{1}{5}$ (d) $\frac{2}{5}$

2. (a) 33 cm (b) 24·75 cm (c) 22 cm (d) 22 cm

3. (a) 10 cm (b) 14 cm (c) 14 cm

4. (a) 150° (b) $103\frac{1}{11}$° (c) 78·75°

4—2. Area of a sector

Let us consider the sector once again and find its area.

In Fig. 4-9, the angle at the centre is 360° and the area of the circle is πr^2.

Can you tell the angle and areas of sectors with the help of the figures ?

Angle	Area
360°	πr^2
180° $\left(\frac{1}{2} \times 360°\right)$	$\frac{1}{2} \pi r^2$
90° $\left(\frac{1}{4} \times 360°\right)$	
45° $\left(\frac{1}{8} \times 360°\right)$	
30° $\left(\frac{1}{12} \times 360°\right)$	

Complete the table. From this we may conclude that the ratio of the angle of the sector to 360° is the ratio of the area of the sector to the area of the circle.

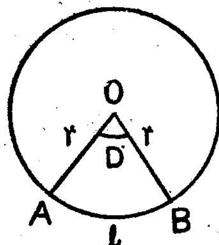
If D is the angle, r the radius and S the area of a sector, then

$$\frac{\text{Area of the sector}}{\text{Area of the circle}} = \frac{D}{360°}$$

$$\text{Area of the sector} = \frac{D}{360} \times \text{Area of the circle}$$

$$\therefore \; S = \frac{D}{360} \times \pi r^2$$

4—3, Perimeter of a sector



The perimeter of the sector OAB = $l + 2r$ where l is the length of the arc and r is the radius.

Fig. 4-13.

Example 1 :

The angle and the radius of a sector are 60° and 21 cm respectively. Find its area and perimeter.

$$\text{Area of the sector} = \frac{D}{360} \times \pi r^2$$

$$= \frac{60}{360} \times \frac{22}{7} \times 21 \times 21$$

$$= 231 \text{ cm}^2$$

$$\text{Length of arc} = \frac{D}{360} \times 2\pi r$$

$$= \frac{60}{360} \times 2 \times \frac{22}{7} \times 21$$

$$= 22 \text{ cm.}$$

$$\text{The perimeter of the sector} = l + 2r$$

$$= 22 + 42$$

$$= 64 \text{ cm.}$$

Example 2 :

The area and the angle of a sector are 44 cm² and 140° respectively. Find its radius.

Area of the sector $= \frac{D}{360} \times \pi r^2$

$$44 = \frac{140}{360} \times \frac{22}{7} \times r^2$$

$$r^2 = \frac{44 \times 360 \times 7}{140 \times 22}$$

$$= 36$$

$$\therefore \quad r = \sqrt{36} = 6$$

The radius of the sector = 6 cm.

4—4. Area of the sector (Aliter)

Length of arc $l = \frac{D}{360} \times 2\pi r$

Area of the sector $= \frac{D}{360} \times \pi r^2$

$$= \frac{D}{360} \times \frac{2\pi r^2}{2}$$

$$= \frac{D}{360} \times 2\pi r \times \frac{r}{2}$$

$$= \frac{l \times r}{2} = \frac{1}{2} l r$$

The area of a sector with l as the length of arc and r as the radius is

$$A = \frac{1}{2} l r$$

Example :

The perimeter of a sector is 52 cm and its radius is 15 cm. Find its area.

Perimeter of the sector $= l + 2r$

$$l + 2 \times 15 = 52$$

$l + 30 = 52$

$l = 52 - 30$

$= 22$

$\therefore$ Length of arc $= 22$ cm

Area of the sector $= \frac{1}{2} l r$

$= \frac{1}{2} \times 22 \times 15$

$= 165$

$\therefore$ Area of the sector $= 165 \text{ cm}^2$

Exercise 4—4 (a)

1. The radius and angle of a sector are 35 cm and 216° respectively. Find its area.

2. Complete the following table where some measures of a sector are given:

	Radius	Length of arc	Angle	Perimeter	Area
i	21 cm	44 cm	...	...	...
ii	21 cm	...	...	97 cm	...
iii	...	88 cm	...	158 cm	...
iv	...	110 cm	300°	...	...

3. Find the angle of the sector of radius 6 cm and area 44 cm^2.

4. Find the area of the shaded portion :

($\sqrt{3}$ = 1·732)

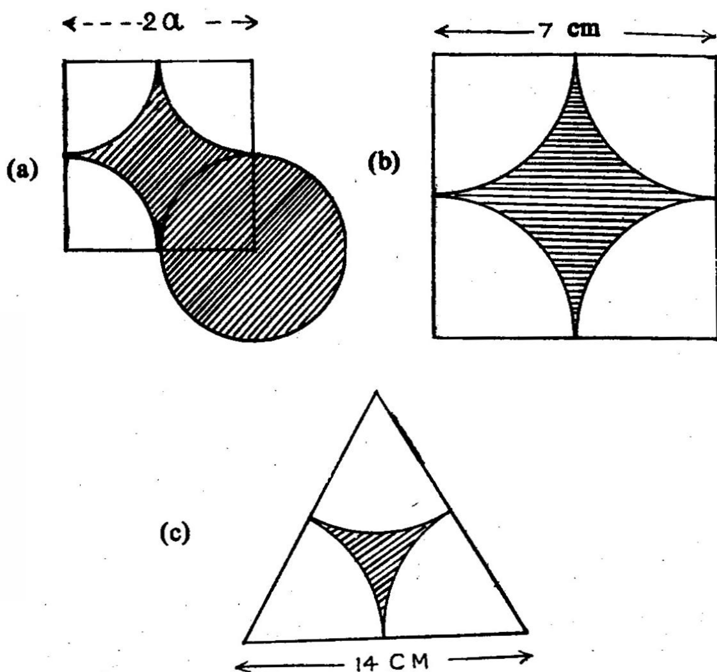


Fig. 4–14.

5. The perimeter and the angle of a sector are 44 cm and 210° respectively. Find its radius and area.

Answers

1. 2310 cm^2 2. (i) 120°, 86 cm, 462 cm^2

(ii) 35 cm, 150°, 577·5 cm^2

(iii) 35 cm, 144°, 1540 cm^2 (iv) 21 cm, 152 cm, 1155 cm^2

3. 140° 4. (a) 4a^2 (b) 10·5 cm^2 (c) 7.868 cm^2

5. 12 cm, 264 cm^2

Exercise 4—4 (b)

1. The radius and the length of an arc of a sector are 8 cm and 13 cm respectively. Find its area.

2. The perimeter and radius of a sector are 48 cm and 15 cm respectively. Find its area.

3. The perimeter and the length of an arc of a sector are 42 cm and 21 cm respectively. Find its area.

4. The area and the length of an arc of a sector are 153 cm^3 and 18 cm respectively. Find its radius.

5. The area of a sector is 68·25 cm^2 and its radius is 13 cm. Find the length of the arc.

6. The area of a sector is 92 cm^2 and its radius is 11·5 cm. Find its perimeter.

Answers

1 52 cm^2 2. 135 cm^2 3. 110·25 cm^2 4. 17 cm

5. 10·5 cm 6. 39 cm

MATHEMATICS CLUB — ACTIVITY 5

1. Everyday at noon a ship leaves New York for England and at the same time another ship leaves England for New York. If the travel time is 144 hours, how many ships from England will meet a ship from New York on its voyage ?

2. Fill the starred places with appropriate numbers in the following multiplication.

		×	×	7
		3	×	×
	×	0	×	×
	×	×	×	
×	5	×		
×	7	×	×	3

5. GEOMETRY

1. Geometrical Terms — Revision

Last year we studied that geometry is the study of set of points. A point is a geometrical notion. The symbol '⊙' is used to denote it. But we know that in reality the point thus marked itself contains many more points and it is only a notation.

We learnt that the sets of points devolop the various geometrical concepts such as line, ray, segment, angle, plane, space etc. Let us recall some of them now.

Name	Figure	Notation	To be read as
Point	·P	P	Point P
Line	l ←——→ M N	l or $\overleftrightarrow{MN}$ or $\overleftrightarrow{NM}$	line l (or) line MN (or) line NM
Segment	X ——— Y	$\overline{XY}$ or $\overline{YX}$	Segment XY or Segment YX
Ray	——→ T V	$\overrightarrow{TV}$	Ray TV

Line, segment and ray are all sets of infinite points. The segment has two end points and the ray has one end point. A line has no end points.

Example 1 :

What is the name of the set of points in Fig. 5-1 ?

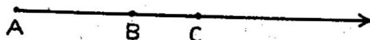


Fig 5-1.

It is a ray with A as the end point. Since B and C are two points on it, this ray can be named $\overrightarrow{AB}$ or $\overrightarrow{AC}$.

Example 2 :

Give all possible names for the line given in Fig. 5-2.

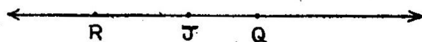


Fig. 5-2.

A line is named by any two of its points. Hence the above line can be named in any of the following ways.

$\overleftrightarrow{RJ}$, $\overleftrightarrow{RQ}$, $\overleftrightarrow{JR}$, $\overleftrightarrow{QR}$, $\overleftrightarrow{JQ}$, $\overleftrightarrow{QJ}$.

Example 3 :

Find the following from the line given in Fig. 5-3.

(a) $\overrightarrow{BA} \cap \overrightarrow{CD}$ (b) $\overrightarrow{BA} \cap \overrightarrow{BC}$ (c) $\overrightarrow{CA} \cap \overrightarrow{BD}$

(d) $\overrightarrow{AC} \cap \overrightarrow{BD}$ (e) $\overrightarrow{CA} \cup \overrightarrow{BD}$

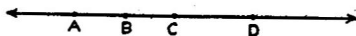


Fig 5-3.

(a) $\overrightarrow{BA}$ and $\overrightarrow{CD}$ have no common point

Hence $\overrightarrow{BA} \cap \overrightarrow{CD} = \phi$

(b) $\overrightarrow{BA}$ and $\overrightarrow{BC}$ have B as common point.

Hence $\overrightarrow{BA} \cap \overrightarrow{BC} = \{ B \}$

(c) The common point of $\overrightarrow{CA}$, $\overrightarrow{BD}$ form the segment $\overrightarrow{BC}$.

Hence $\overrightarrow{CA} \cap \overrightarrow{BD} = \overline{BC}$

(d) $\overrightarrow{AD}$ is another name of $\overrightarrow{AC}$.

Hence $\overrightarrow{AC} \cap \overrightarrow{BD} = \overrightarrow{AD} \cap \overrightarrow{BD} = \overrightarrow{BD}$

(e) With C as the end point $\overrightarrow{CA}$ moves towards the left, whereas with B as the end point $\overrightarrow{BD}$ moves towards the right. The set of common points of these two form the line $\overleftrightarrow{AD}$.

Hence $\overrightarrow{CA} \cup \overrightarrow{BD} = \overleftrightarrow{AD}$

Angle:

The union of two rays having the same end point is called an angle. Fig. 5-4 represents an angle.

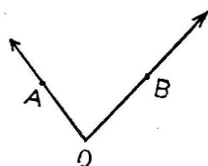


Fig. 5-4.

$\overrightarrow{OA} \cup \overrightarrow{OB} = \angle AOB = \angle BOA$.

In Fig. 5-5 $\overrightarrow{AB}$, $\overrightarrow{CD}$ do not form an angle as they do not have a common end point.

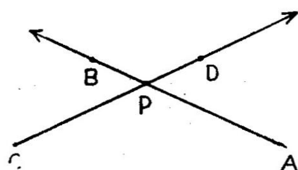


Fig. 5-5.

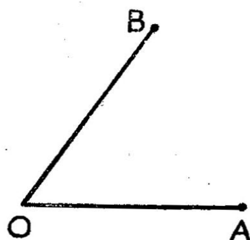


Fig. 5-6.

In Fig. 5-6, $\overline{OA} \cup \overline{OB}$ does not form an angle. Why?

$\overline{OA}$ and $\overline{OB}$ are segments. The union of any two segments does not form an angle.

Recall:

An angle is the union of two rays with a common end point.

Exercise 1

1. (a) Give three possible names for the line in Fig. 5-7.

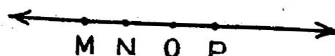


Fig. 5-7.

(b) Mention three rays in it.

(c) Mention three segments.

(d) Give a simple name for each of the following:

(i) $\overline{MO} \cup \overline{NP}$ (ii) $\overline{MN} \cap \overline{NP}$ (iii) $\overrightarrow{OM} \cap \overrightarrow{NP}$

(iv) $\overleftrightarrow{NO} \cup \overrightarrow{OP}$ (v) $\overline{MO} \cap \overrightarrow{NP}$ (vi) $\overrightarrow{NM} \cap \overline{OP}$

(vii) $MP \cup \overline{NO}$ (viii) $\overline{MN} \cup \overline{OP}$ (ix) $\overline{OP} \cap \overrightarrow{MN}$

(x) $\overrightarrow{OM} \cup \overrightarrow{NP}$

2. State whether the following are true or false. Draw figures to explain the same.

(a) The union of two rays can be a line.

(b) The union of two rays can be an angle.

(c) Two lines will intersect at two and only two points.

(d) The union of two segments is a line.

3. Draw figures to explain the following:

(a) Two segments in a line.

(b) A segment and a ray, their union and intersection.

(c) Two segments not on the same line.

(d) Two rays having the same end point.

Answers

1. (d) (i) $\overline{MP}$ (ii) $\{N\}$
2. (a), (b) True; (c), (d) false

2. Plane

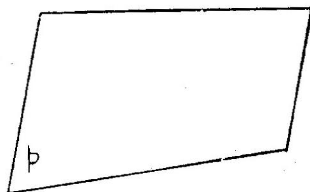


Fig. 5-8.

Fig. 5-8 represents a plane. A plane is a set of points. A line has no end points. Like that a plane has no ends. It can be extended in all directions. A plane contains an infinite number of lines. We know that a line is determined by two points. To fix a plane we need one more point which is not in line with the above two points. Hence a set of three non-collinear points determine a plane. Since two points determine a line we can say that a line and a point not on it, determine a plane.

e.g. $\overleftrightarrow{AB}$, C ; $\overleftrightarrow{AC}$, B ; $\overleftrightarrow{BC}$, A

With the same three points we can form the lines $\overleftrightarrow{AB}$, $\overleftrightarrow{AC}$, $\overleftrightarrow{BC}$.

·B

A· ·C

Fig. 5-9.

A plane can be expressed in terms of the pairs of lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$ or $\overleftrightarrow{AB}$ and $\overleftrightarrow{BC}$ or $\overleftrightarrow{AC}$ and $\overleftrightarrow{BC}$.

Hence a plane is determined by any one of the following.

(1) Three non-collinear points.

(2) A line and a point not on it.

(3) Two intersecting lines

Exercise 2

1. Find the following from the given figure.

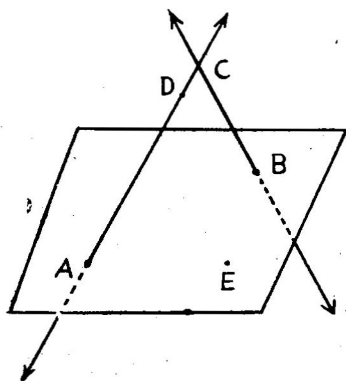


Fig. 5-10.

(a) Three collinear points

(b) Three coplanar points

(c) Three non-collinear points

(d) Three non-coplanar points.

2. Find the answers through experiments.

(a) The intersection of a plane and a line not on it.

(b) The intersection of two planes.

3. If Fig. 5-11 represents a solid, identify the sets of points given below as collinear, coplanar etc.

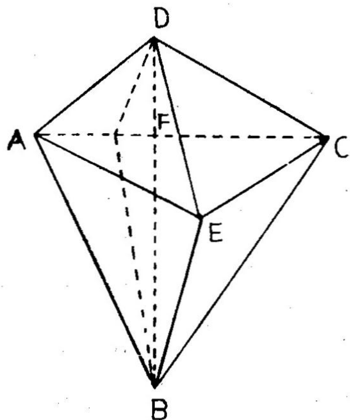


Fig. 5-11.

$\overline{AC} \cap \overline{BD} = \{F\}$

(a) {D, E}

(b) {A, B, E}

(c) {A, E, C}

(d) {A, B, C, D}

(e) {B, D, F}

(f) {B, C, D, E}

(g) {A, D, C, E}

(h) {A, F, C}

3. Closed curves, polygons

3—1. Clased curves

Look at the following curves :

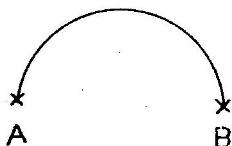


Fig. 5-12.

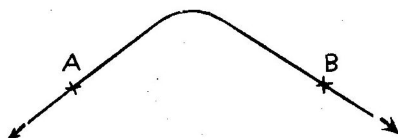


Fig. 5–13.

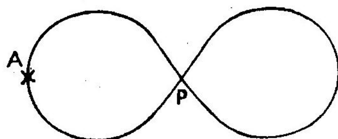


Fig. 5–14.

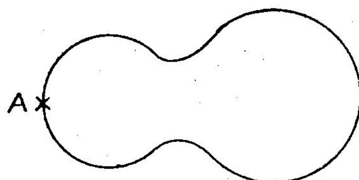


Fig. 5–15.

The curve in Fig. 5-12 has two end points.

The curve in Fig. 5-13 does not have two end points.

In Figures 5–14 and 5-15 any point can be taken as the end point. Observe that we can start from any point and reach the same point after tracing the curve. Such curves are called closed curves. We learn that Fig. 5-13 divides the plane into two parts. These two parts have no ends. In Figures 5-14 and 5-15 the plane is divided into three parts.

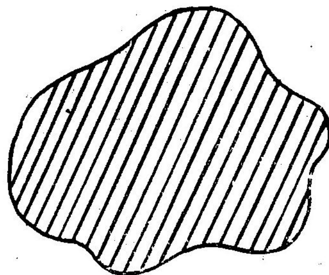


Fig. 5-16.

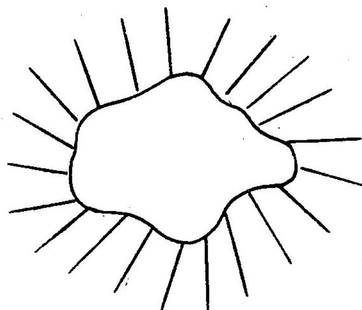


Fig. 5-17

Note the following:

(1) The **interior** of the curve (Fig. 5-16)

(2) The **exterior** of the curve (Fig. 5-17)

(3) The **boundary** of the curve (Fig. 5-14 and 5-15)

The closed part contains the boundary and the interior.

In Fig. 5-14, if we start from A and move along the boundary, we will cross P twice before reaching A. But in Fig. 5-15 we cannot cross so. Such curves are called **simple closed curves**. Since the French Mathematician Carmile Jordan (1838 - 1922) defined such curves, they are called Jordan curves. On looking at the figures, we can recall some of the simple closed curves studied so far. Circle, triangle and qnadrilateral are all simple closed curves. Have we not found out the perimeter and area of these figures ? Now we know that the perimeter is the length of the boundary and the area is connected with the interior of the curve.

3—2. Polygons

Figures like triangle, square, quadrilateral, parallelogram, rectangle and hexagon are simple closed curves formed by segments. Their general name is **polygon**. Their names correspond to the number of angles in them. The fact that they are simple closed curves has a greater mathematical implication.

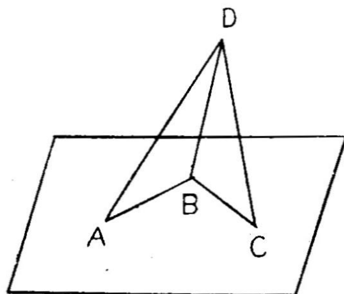


Fig. 5-18.

We can learn from Fig. 5-18 that four segments need not necessarily form a quadrilateral. Observe that the segments $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{AD}$ are not coplanar. It is important to note that polygons are plane figures.

In polygons each segment is called a side and the common point of two segments is called the vertex. We know that an angle is the union of two rays. Since polygons have only segments their angles must be considered as the union of the rays got by extending the segments.

Example :

In Fig. 5-19, $\angle ABC$ is found by $\overrightarrow{BA}$ and $\overrightarrow{BC}$.

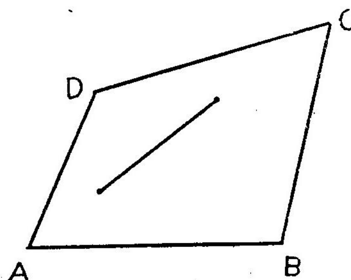


Fig. 5-19.

Similarly $\overrightarrow{AD} \cup \overrightarrow{DC} = \angle ADC$, $\overrightarrow{DC} \cup \overrightarrow{CB} = \angle DCB$ and $\overrightarrow{AB} \cup \overrightarrow{AD} = \angle DAB$.

No point of the rays $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CD}$ and $\overrightarrow{DA}$ are in the interior of the polygon. Such polygons are called convex polygons.

In Fig. 5-20 $\angle AED$ is the union of $\overrightarrow{ED}$ and $\overrightarrow{EA}$. Some points of $\overrightarrow{ED}$ are located in the interior of the polygon.

Similarly a portion of $\overrightarrow{CD}$ lies in the interior of the polygon. Such polygons are called concave polygons. It can be observed that all the points of a segment obtained by joining two points in the interior of a

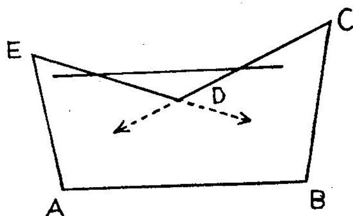


Fig. 5-20.

convex polygon lie within the polygon (Fig. 5-19). On the other hand, we can find two points such that some points of the segment joining them lie in the exterior of the polygon (Fig. 5-20)

3—3. Diagonals

The segment joining two non-consecutive vertices of a polygon is called a **diagonal**. In Fig. 5-21, B and E are adjacent to A ; C and D are not adjacent to A. $\overline{AC}$, $\overline{AD}$ are diagonals; $\overline{BD}$, $\overline{BE}$, CE are also diagonals.

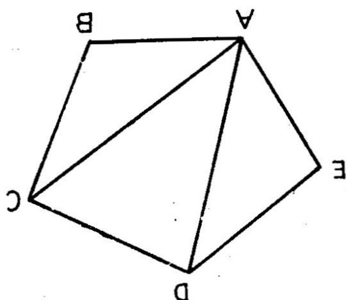


Fig. 5-21.

In a convex polygon all the points of the diagonals lie in the interior of the polygon. Verify whether it is true with respect to a concave polygon.

Draw polygons having 3, 4, 5, 6, 7 and 8 sides. Find out the number of diagonals in each case. Tabulate them. Hereafter let us assume that polygons will mean only convex polygons.

As shown in Fig. 5-21 we know that the diagonals drawn from a vertex of a pentagon divide it into 3 triangles and the sum of the angles of the triangles is $3 \times 180° = 540°$. Hence the sum of the interior angles of a pentagon is 540°. Similarly find the sum of the interior angles of the polygon having 6, 7, 8 sides. Let us tabulate the results.

Sides	Diagonals	Triangles	Sum of the angles
3	0	1	180°
4	2	2	360°
5	5	3	540°
6	9	4	720°
7	14	5	900°
8	20	6	1080°
9	?	?	?
10	?	?	?

Verify whether we can denote the number of diagonals of a polygon having 'n' sides as $\frac{n(n-3)}{2}$ and the sum of the angles 180 (n — 2).

3–4. Angles of a polygon

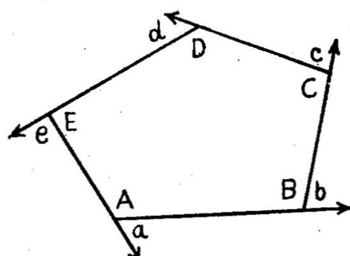


Fig. 5-22.

In Fig. 5-22 we have shown the rays following the angles (anticlockwise) of a polygon. These rays form angles outside the polygon. If we denote the interior angles as A, B, C, D, E and the exterior angles as a, b, c, d, e then we can find that $m\angle A + m\angle a = 180°$, $m\angle B + m\angle b = 180°$ and so on.

Hence the sum of the interior and the exterior angles of a pentagon is $5 \times 180° = 900°$.

The sum of the interior angles $= 540°$.

Hence the sum of the exterior angles is $900° - 540° = 360°$.

Similarly find the sum of the exterior angles of the polygons having 6, 7, 8 sides. What do you find?

The sum of 'n' interior angles $= 180\,(n - 2)$

$$= 180\,n - 360$$

The sum of 'n' interior + exterior angles $= 180\,n$.

Hence the sum of 'n' exterior angles $= 180n - (180n - 360)$

$$= 180\,n - 180\,n + 360 = 360°.$$

If a polygon has equal sides and equal interior angles then it is called a **regular polygon.** Some of the regular polygons known to us are, equilateral triangles and squares.

Let us now try to find the interior angles of a regular polygon.

The sum of the interior angles of an 'n' polygon is $180\,(n - 2)$. Hence the interior angles of a regular n-gon is $\frac{180}{n}(n - 2) = 180 - \frac{360}{n}$

The exterior angle of a regular n-gon $= \frac{360}{n}$.

Example :

Find the interior angle of a regular decagon.

A decagon has 10 sides and 10 interior angles.

One exterior angle $= \frac{360}{10} = 36°$

Interior angle $= 180° - 36° = 144°$.

Exercise 3

1. Can we have a two sided polygon? If not, why?

2 In Fig. 5-23 start from any point on the curve and after moving along the curve see whether you come back to the same point. If so it is a closed curve.

Take one point in the interior and one in the exterior of the

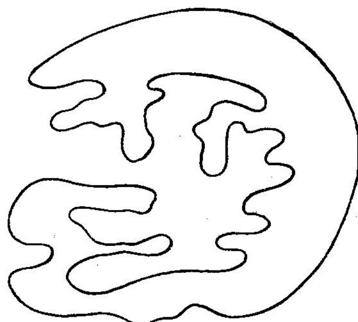


Fig. 5-23.

curve and join them. In how many points does the line cut the curve? What can we infer from this? (This is known as Jordan Holder Theorem). When closed curves such as the one found in Fig. 5-23 are given, we can find whether a point is in the interior or in the exterior of it by making use of the above theorem.

3. See whether it will hold good for all closed curves.

4. Will the interior angle of a regular polygon exceed 180°? Will it be equal to it?

5. Find the interior as well as the exterior angles of the regular polygons having 12, 18, 24 and 30 sides.

6. What is the number of sides of a regular polygon having 179° as the interior angle?

7. The vertices of certain regular polygons can be so arranged that they coincide at a single point. For example 6 equilateral triangles can be placed as shown in Fig. 5-24. What are the other regular polygons that can be fixed in this way?

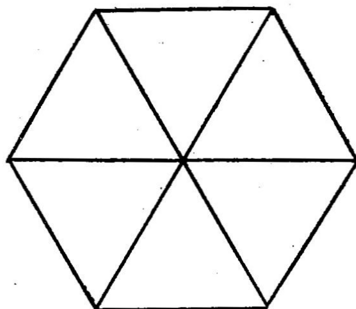


Fig. 5-24.

8. In the last sum we dealt with the same kind of regular polygons coinciding at a point. We can also make use of different kinds of regular polygons for this. For example,

(1) 3 triangles and 2 squares.

(2) 2 triangles and 2 hexagons.

(3) 2 triangles, one do-decagon and one sqnare. Similarly if (a) a heptagon (b) a decagon (c) one do-decagon are given, find out the regular polygons that can be attached to them to cover an area without overlapping. Find out the sum of the angles of the figures given in the example.

Answers

(1) A closed curve cannot be formed with two sides.

(4) Will be less than 180°.

(5) (i) 150°, 30° (ii) 160°, 20°

(iii) 165°, 15° (iv) 168°, 12°

4. Locus of Points

The set of points when a point traverses under certain specified conditions is called the locus of that point. Let us recall some of the loci we have studied earlier. (Loci is plural of locus.)

4—1. The locus of points equidistant from a given point

We saw that points which are equidistant from a fixed point form a circle. The fixed point is the centre and the given distance is the radius.

Hence the locus of points at a given distance from a given point is a circle.

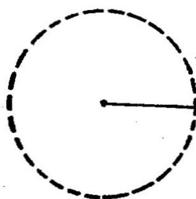


Fig. 5-25.

What is the locus of a point which moves in such a way that it is always equidistant from two given points ? Let us take the distance between the two given points to be 5 cm and the fixed distance to be 3 cm. The locus of points moving at a distance of 3 cm from A is the circle I with centre A and radius

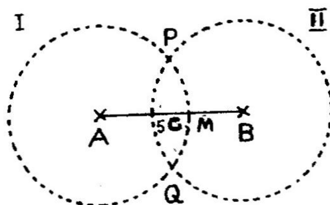


Fig. 5-26.

3 cm. The locus of points moving at a distance of 3 cm from B is the circle II with centre B and radius 3 cm. The points at a distance of 3 cm from A and B are the two intersecting points P and Q of the circles I and II.

Hence the locus of points moving at a fixed distance from two given points is the set of two points.

Find the relationship between $\overline{PQ}$ and $\overline{AB}$. A and B are the two points which are at a distance of 3 cm from P and Q.

PQ is the perpendicular bisector of $\overline{AB}$.

If the fixed distance is 2 cm, can we find P, Q ? What do we infer from this ?

4—2. The locus of points equidistant from two given points

In the last lesson we learnt that the locus of points moving at a fixed distance from two given points is the set of two

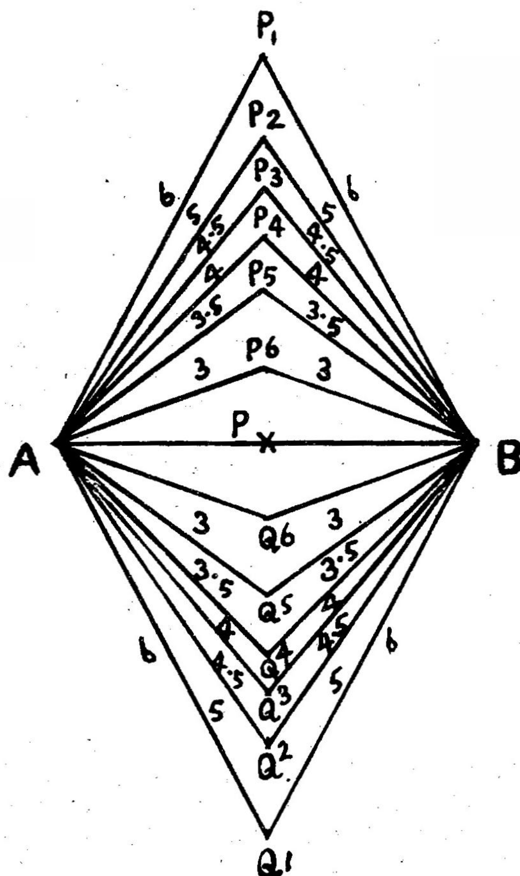


Fig. 5-27.

points lying on the perpendicular bisector of the segment joining the two given points.

Let us now make a change in the condition. Instead of a fixed distance, let us state it as equidistant. The points are

fixed. But the moving points are equidistant from the two given points. In the last lesson the fixed distance, was 3 cm. Instead, if we call it as equidistant the equal distances can be taken as 6 cm, 5 cm, 4·5 cm, 4 cm, 3·5 cm, 3 cm, 2·5 cm etc. P_1, Q_1 are at a distance of 6 cm from A, B; P_2, Q_2 are at a distance of 5 cm; P_3, Q_3 are at a distance of 4·5 cm. Similarly (P_4, Q_4), (P_5, Q_5), (P_6, Q_6) are at distances of 4 cm, 3·5 cm, 3 cm respectively. We know that $\overline{P_1 Q_1}$, $\overline{P_2 Q_2}$, $\overline{P_3 Q_3}$, $\overline{P_4 Q_4}$, $\overline{P_5 Q_5}$, $\overline{P_6 Q_6}$ are the perpendicular bisectors of $\overline{AB}$.

Hence the locus of a point equidistant from two given points is the perpendicular bisector of the segment joining the two points.

This can be verified through paper folding. Draw a segment joining two points on a sheet of paper. Fold the paper in such a way that the two points coincide. The fold is the perpendicular bisector of the segment. Measure the distances of the points on the segment from the two given points.

4—3. Point moving equidistant from two parallel lines

Let us find out the locus of a point moving equidistant from two given parallel lines. Take a ruled sheet and select any two of the parallel lines. Fold the paper in such a way that the two parallel lines coincide. Are not the points on the fold equidistant from the two parallel lines ? The fold is also a parallel line running mid-way between them.

In Fig. 5-28, $\overleftrightarrow{AB}$, $\overleftrightarrow{CD}$ are two parellel lines. $\overleftrightarrow{PQ}$ is the locus of the points equidistant from them.

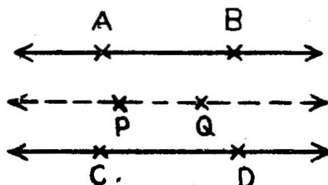


Fig 5-28.

Now if we take $\overleftrightarrow{PQ}$ as a fixed line, then can't we take $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ as the loci of the points equidistant from it ?

Do some more experiments and verify the validity of the above statement. From this we know,

(1) The locus of a point equidistant from two given parallel lines is a line parallel to each of the given lines and mid-way between them.

(2) The loci of points at a given distance from a given line is a pair of lines each parallel to the given line and at the given distance from it on either side of the given line.

4—4. The locus of a point equidistant from two intersecting lines

Draw an angle on a piece of paper. Fold the paper such that the rays of the angle coincide.

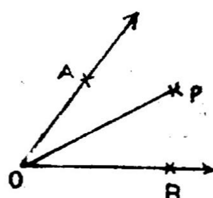


Fig. 5-29.

Is not the fold bisecting the angle ?

OP falls along the fold and bisects $\angle AOB$.

Mark points on $\overrightarrow{OP}$.

From this we know that the locus of the point equidistant from the rays having a common end point is the bisector of the angle formed by the rays.

Draw two intersecting lines. There are four angles at the intersecting point. The loci of the points equidistant from them are the bisectors of the angles.

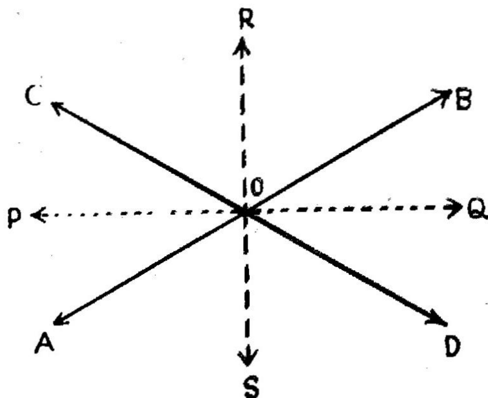


Fig. 5-30.

Draw two intersecting lines on a piece of paper and find the bisectors of the angles by folding the paper properly.

The points on $\overrightarrow{OP}$, $\overrightarrow{OQ}$ lie on a straight line. Similarly the points on $\overrightarrow{OR}$, $\overrightarrow{OS}$ lie on another straight line. We can see that $\overrightarrow{PQ}$ and $\overrightarrow{RS}$ are at right angles.

Hence the loci of points which are equidistant from the two intersecting lines are the two bisectors of the angles formed by the segments of the lines. The bisectors are also perpendicular to each other.

Exercise 4

Define the loci of the following with proper diagrams:

1. Points which are 3 cm away from a fixed point

2. Points which are 3 cm away from a fixed line

3. Points which are equidistant from a pair of parallel lines 5 cm apart

4. Points which are equidistant from the arms of an angle of 60°

5. Points which are at a distance of 4 cm from the points P and Q which are 6 cm apart.

Answers

1. Diameter.

2. Parallel lines.

3. Parallel line.

4. Bisector of the angle.

5. Perpendicular bisector of PQ.

5—1 (i) Parallelogram (a)

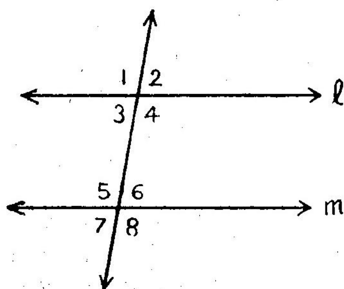


Fig. 5-31.

If $l \parallel m$,

$\angle 3 \cong \angle 6$; $\angle 4 \cong \angle 5$

$\angle 1 \cong \angle 4$; $\angle 1 \cong \angle 5$

$\angle 2 \cong \angle 3$; $\angle 2 \cong \angle 6$

$\angle 3 \cong \angle 7$; $\angle 4 \cong \angle 8$

A parallelogram is a quadrilateral whose opposite sides are parallel.

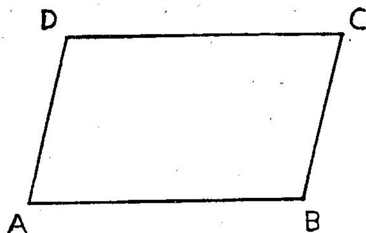


Fig. 5-32.

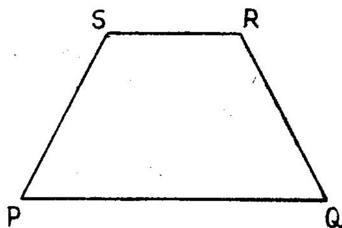


Fig. 5-33.

In Fig. 5-32, AB $\parallel$ DC; AD $\parallel$ BC

ABCD is a parallelogram.

In Fig. 5-33, PQ $\parallel$ SR. Is this a parallelogram ? If not, why?

Since the opposite edges of a scale are parallel draw a parallelogram with it and take the cut out. Fix side DC on AB. See whether they are congruent.

Similarly see that $\overline{AD} \cong \overline{BC}$

Cut the parallelogram along BD. The two triangles thus got will be congruent.

$\triangle ABD \cong \triangle CDB$

$\therefore$ $\angle A \cong \angle C$. Similarly we can see that $\angle B \cong \angle D$.

Take another parallelogram and fold it along AC and BD. We get four segments. See whether any two of them are congruent. From this experiment we learn that

> In a parallelogram,
>
> (i) Opposite sides are parallel
>
> (ii) Opposite sides are congruent
>
> (iii) Opposite angles are congruent
>
> (iv) Diagonals bisect each other

Construct a number of parallelograms, and measure their sides, angles and segments caused by the intersection of the diagonals and verify the above statements.

Example :

PQRS is a parallelogram (Fig. 5-34).

$m\angle S = 4x - 60$; $m\angle Q = 30 - x$. Find the angles.

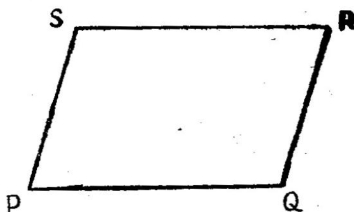


Fig. 5 - 34.

$m\angle S = m\angle Q$ (opposite angles)

$\therefore\ 4x - 60 = 30 - x$

$4x + x = 30 + 60$

$5x = 90$

$x = 18$

$m\angle S = 4x - 60 = 72 - 60 = 12^\circ$

$m\angle Q = 12^\circ$

$m\angle P = m\angle R = 180^\circ - 12^\circ = 168^\circ$

Exercise 5—1 (i)

1. JKLM is a parallelogram. Find out the equal sides and equal angles.

2. In parallelogram RSTU, $m\angle R = 130°$. Find the remaining angles.

3. In parallelogram SBCJ, if $m\angle B = 2\, m\angle S$, find $m\angle S$ and $m\angle B$.

4. In parallelogram SBCJ, $m\angle S = 3x - 5$; $m\angle B = 4x - 25$. Find $m\angle S$ and $m\angle B$.

5. In parallelogram PWJL, $m\angle P = 7x - 12$; $m\angle W = 2x + 3$. Find $m\angle P$, $m\angle W$.

6. In parallelogram ABCD, $AB = 5x - 3$; $DC = 3x + 3$. Find the measure of AB.

7. ABCD is a parallelogram. Which of the following are congruent ? State the reason.

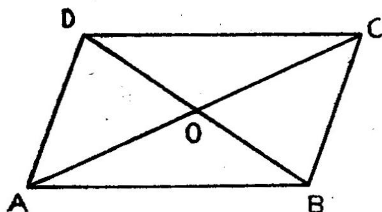


Fig. 5 - 35.

(a) △ AOD, △ COB
(b) △ ABC, △ ABD
(c) △ ADB, △ CDB
(d) △ AOB, △ BOC

8. In Fig 5 - 35, OA = 3 cm, OB = 2·5 cm, AB = 4 cm. Find AC, BD, DC.

Answers

4. $m\angle S = 85$, $m\angle B = 95$

6. AB = 12   7. (a), (c) congruent.

5—1 (ii) Parallelogram (b)

Let us prove mathematically all we have studied in the last lesson about the parallelogram.

In the last lesson, we learnt that if, in $\triangle$ ABC and $\triangle$ DEF,

(1) $AB = DE$; $BC = EF$; $CA = FD$ (SSS) or

(2) $AB = DE$; $BC = EF$; $m\angle B = m\angle E$ (SAS) or

(3) $AB = DE$; $m\angle A = m\angle D$; $m\angle B = m\angle E$ (SAA)

then $\triangle ABC \equiv \triangle DEF$.

If $\triangle ABC \equiv \triangle DEF$, then $AB = DE$; $AC = DF$; $BC = EF$; $m\angle A = m\angle D$; $m\angle B = m\angle E$; $m\angle C = m\angle F$

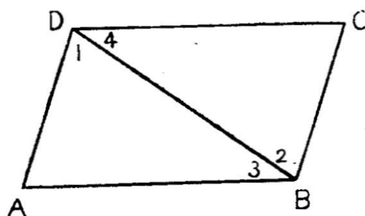


Fig. 5-36.

In parallelogram ABCD,

$AB \parallel DC$, $AD \parallel BC$.

Join BD.

In $\triangle$ ABD and $\triangle$ CDB

(i) $\overline{BD} \equiv \overline{DB}$ (common side)

(ii) $\angle 1 \equiv \angle 2$ ($AD \parallel BC$, DB transversal)

(iii) $\angle 3 \equiv \angle 4$ ($AB \parallel DC$, DB transversal)

$\therefore$ $\triangle ABD \equiv \triangle CDB$ (SAA)

$\therefore$ $\overline{AB} \equiv \overline{DC}$; $\overline{AD} \equiv \overline{BC}$; $\angle A \equiv \angle C$

Similarly it can be proved that $\angle B \equiv \angle D$

In a parallelogram the opposite sides are congruent and the opposite angles are congruent.

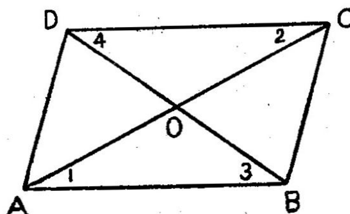


Fig. 5-37.

In $\triangle$ OAB and $\triangle$ OCD,

$\overline{AB} \equiv \overline{DC}$ (opposite sides)

$\angle 1 \equiv \angle 2$ (why ?)

$\angle 3 \equiv \angle 4$ (why ?)

$\therefore$ $\triangle$ OAB $\equiv$ $\triangle$ OCD (SAA)

Hence $\overline{OA} \equiv \overline{OC}$; $\overline{OB} \equiv \overline{OD}$ or AC, BD bisect each other.

In a quadrlateral,

(i) If the opposite sides are cangruent or,

(ii) If the opposite angles are congruent or,

(iii) If a pair of opposite sides are parallel and congruent or

(iv) If the diagonals bisect each other

then the quadrilateral is a parallelogram.

Prove these statements making use of the congruency of triangles.

Exercise 5—1 (ii)

1. In quadrilateral ABCD the diagonals $\overline{AC}$, $\overline{BD}$ intersect at O. AO = CO = 8; BO = DO = 7. What kind of a quadrilateral is it ?

2. **State with reasons, whether ABCD is a parallelogram for each of the following data.**

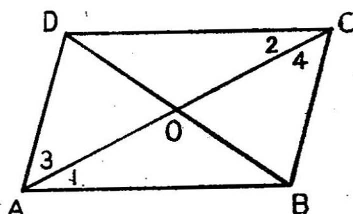


Fig. 5-38.

(a) $\overline{AB} \parallel \overline{DC}$; $AD \parallel \overline{BC}$

(b) $\overline{AB} \cong \overline{DC}$; $\overline{AD} \cong \overline{BC}$

(c) $AB \parallel \overline{DC}$; $\overline{AD} \cong \overline{BC}$

(d) $\overline{DO} \cong BO$; $\overline{AO} \cong \overline{CO}$

(e) $\triangle ABC \cong \triangle ADC$

(f) $\angle 1 \cong \angle 2$; $\angle 3 \cong \angle 4$

(g) $\angle ABC \cong \angle ADC$; $\angle DAB \cong \angle DCB$

(h) $\overline{AC} \cong \overline{BD}$

(i) $\triangle AOB \cong \triangle COD$

(j) $\overline{AB} \cong \overline{DC}$; $\angle 2 \cong \angle 1$

3. In a quadrilateral ABCD, $\angle A$ and $\angle B$ are supplementary angles. $\angle A$ and $\angle D$ are also supplementary angles. State whether ABCD will constitute a parallelogram. Give reason.

4. Are the following quadrilaterals parallelograms ?

(a)

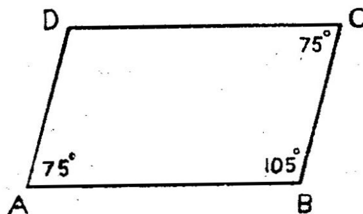


Fig. 5-39.

(b)

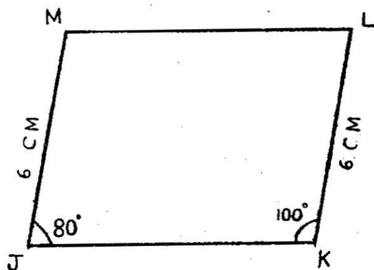


Fig. 5-40.

(c)

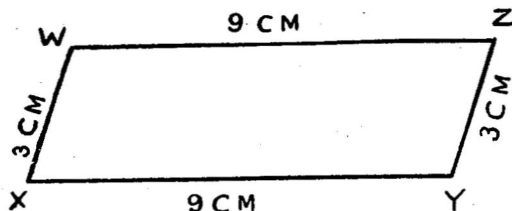


Fig. 5-41.

Answers

1. Parallelogram
2. Yes. In a parallelogram the adjacent angles are supplementary.
4. (a), (b), (c) Yes

5—2. Triangles

Cut off a triangle from a sheet of paper. The mid point D of $\overline{AB}$ can be got by folding the paper in such a way that A and C coincide. Similarly find the mid point E of AC. See whether $\overline{DE}$ is parallel to $\overline{BC}$. Also verify that $\overline{DE} = \frac{1}{3}$ BC.

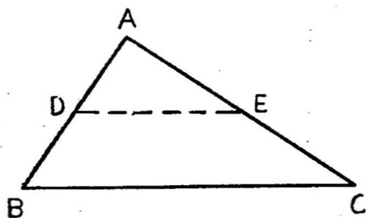


Fig. 5-42.

Form triangles with the help of match sticks and verify the above statements.

> The segments got by joining the mid points of any two sides of a triangle is parallel to the third side and is half of that.

Let us prove it mathematically.

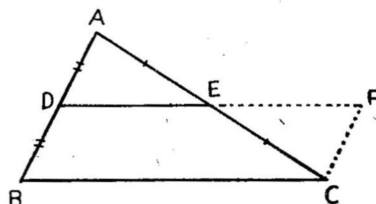


Fig. 5 – 43.

Mark F such that D, E, F are collinear and $\overline{DE} \equiv \overline{FE}$.

Draw $\overline{FC}$.

In $\triangle$ AED and $\triangle$ CEF

$\overline{AE} \equiv \overline{EC}$ (E is the midpoint of AC)

$\overline{DE} \equiv \overline{EF}$ (construction)

$\angle AED \equiv \angle CEF$ (vertically opposite angles)

Hence $\triangle$ AED $\equiv$ $\triangle$ CEF (SAS)

Hence $\overline{CF} \equiv \overline{AD} \equiv \overline{DB}$

$\angle ECF \equiv \angle DAE$ $\therefore$ $\overline{AD} \parallel \overline{FC}$

$\therefore$ $\overline{DB} \parallel \overline{FC}$; $\overline{DB} \equiv \overline{FC}$;

$\therefore$ BDFC is a parallelogram

$\therefore$ $\overline{DF} \parallel \overline{BC}$ or $\overline{DE} \parallel \overline{BC}$

$\overline{DF} = \overline{BC}$; DF = 2 DE

$2\,\overline{DE} = \overline{BC}$; $\overline{DE} = \frac{1}{2}\,\overline{BC}$

Examples :

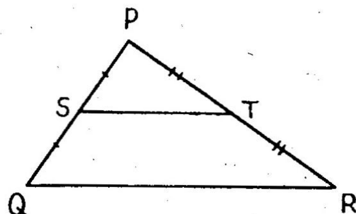


Fig. 5 - 44.

(1) In $\triangle$ PQR, S is the mid point of $\overline{PQ}$; T is the mid point of PR. If ST = 7, find QR.

QR = 2 ST = 14.

(2) The segments joining the mid points of the sides of a quadrilateral form a parallelogram.

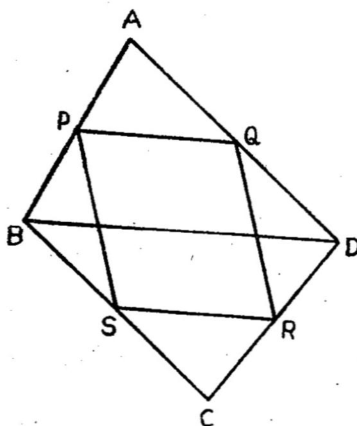


Fig. 5 - 45.

In $\triangle$ ABD, P is the mid point of $\overline{AB}$;

Q is the mid point of $\overline{CD}$

Hence $\overline{PQ} \parallel \overline{BD}$ and PQ $= \frac{1}{2}$ BD

Similarly in $\triangle$ BCD, $\overline{SR} \parallel \overline{BD}$ and SR $= \frac{1}{2}$ BD

$\overline{PQ}$ and $\overline{SR}$ are both parallel to $\overline{BD}$

$\therefore$ $\overline{PQ} \parallel \overline{SR}$; PQ $= \frac{1}{2}$ BD $=$ SR

$\therefore$ PQRS is a parallelogram [opposite sides are equal and parallel]

Exercise 5—2

1. In $\triangle$ ABC, K is the mid point of $\overline{AB}$ and L that of $\overline{AC}$.

   (a) If BC = 10 find KL

   (b) If BC = 3 find KL

(c) If KL = 6 find BC

(d) If KL = 7, find BC

2. In Fig. 5-46, L, M and N are the mid points of $\overline{AB}$, $\overline{AC}$ and $\overline{BC}$ respectively.

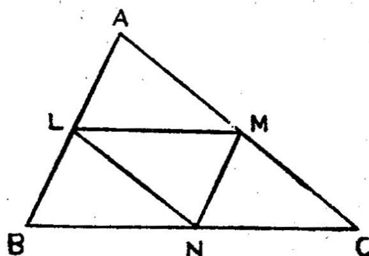


Fig. 5-46.

(a) LN = 12 ; AC = ?

(b) MN = 2x ; AB = ?

(c) LN ∥ ?

(d) BN ∥ ?

(e) BN = 3 ; LM = ?

(f) AL = p ; MN = ?

3. In Fig. 5-46,

(a) BC = 4x — 6 ; LM = x — 1 ; LM = ?

(b) AB = 5m — 2 ; MN = 2x + 3 ; AB = ?

(c) How many parallelograms are there ?

4. Prove that the segments joining the mid points of the opposite sides of a quadrilateral bisect each other.

Answers

1. (a) 5 (b) $1\frac{1}{2}$ (c) 12 (d) 14

2 (a) 24 (b) 4x (f) p

5—3. Three Parallel Lines

Take a ruled sheet of paper having parallel lines at equal intervals. Draw a transversal. See whether the segments cut off by the parellel lines are equal. Draw another transversal and see whether the segments are equal. Draw some more transversals and measure the lengths of the segments. From this experiment we infer the following fact :

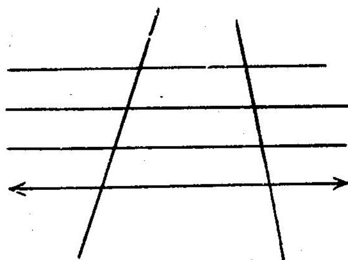


Fig. 5-47.

If parallel lines cut off congruent segments on one transversal, then they cut off congruent segments on any other transversal.

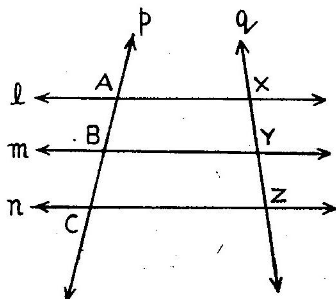


Fig. 5-48.

Note carefully :

$l \parallel m \parallel n$

line p intersects them at A, B and C;

line q intersects them at X, Y and Z.

If $\overline{AB} \equiv \overline{BC}$, then $\overline{XY} \equiv \overline{YZ}$

$\overline{XY}$ need not be equal to $\overline{AB}$

If AB = BC = 5 cm and XY = 6 cm, then YZ = 6 cm.

This principle has been made use of in dividing a line into equal segments.

Exercise 5—3

1. In Fig. 5-49, $l \parallel m \parallel n$; AD = DG = 5

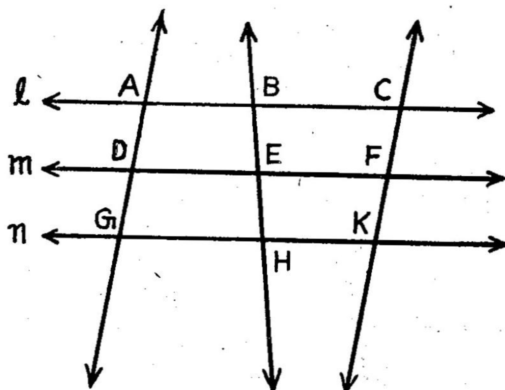


Fig. 5-49.

(a) BE = 4; EH = ?

(b) EH = 3; BE = ?

(c) CF = 10; CK = ?

(d) BH = 9; EH = ?

(e) BH = 14; BE = ?

(f) CK = 11; CF = ?

(g) CF = 7; FK = ?

(h) CK = 13; FK = ?

2. In the above figure if

(a) BE = 3x + 4; EH = x + 8, find BE.

(b) CF = 3y — 2; CK = 3y + 8; FK = ?

(c) AD = 2x + 5; DG = 4x — 3; AG = ?

3. Say whether the following statements are true or false. If found false, give a counter example to prove it is not true.

(a) If three lines cut off congruent segments on each of two transversals, then the lines are parallel.

(b) If a line drawn parallel to one side of a triangle divides the second into two congruent segments, then it will divide the third side also into two congruent segments.

4. In Fig. 5-50, $a \parallel b \parallel c \parallel d \parallel e \parallel f$

$\overline{AB} = \overline{BC} = \overline{CD} = \overline{DE} = \overline{EF}$

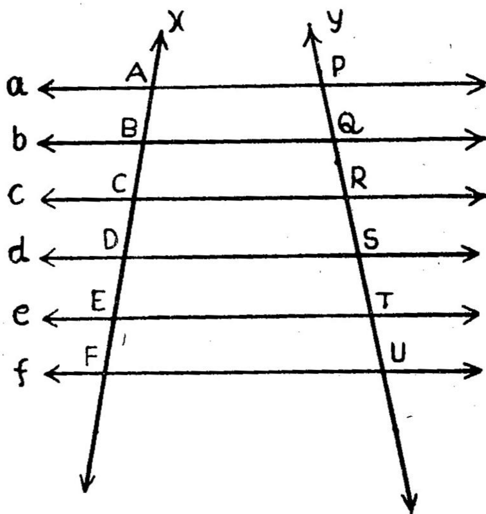


Fig. 5-50.

(a) PQ = 6; TU = ?; PU = ?

(b) QS = 8; PR = ?

(c) AC = 10; DE = ?

(d) PS = 15; RU = ?

5. In Fig. 5-51, if $\overline{AB} \cong \overline{BC}$; $\overline{AH} \parallel \overline{BG} \parallel \overline{CF}$;

$\overline{LF} \parallel \overline{KG} \parallel \overline{JH}$; prove that $\overline{JK} \cong \overline{KL}$

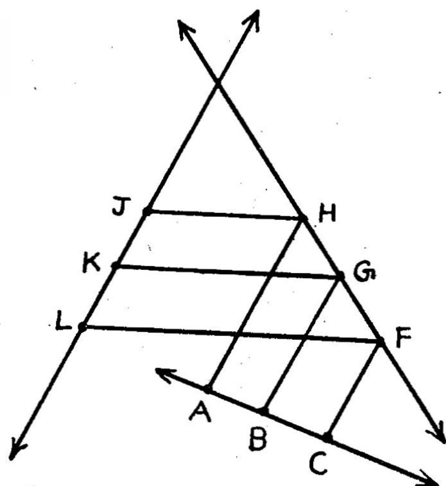


Fig. 5-51.

6. Circles

6—1. General Geometric Terms

We have already learnt about the centre, radius and chord of a circle. Let us learn some more facts about circles.

A circle is a simple closed curve. Hence it has an interior, an exterior and a boundary. The interior including the boundary is called the **circular portion.** The segment joining any two points on the circle is called a **chord.** The chord passing through the centre is called the **diameter.** Any line intersecting a circle at two points is called a **secant.** The secant makes a chord. A line intersecting the circle exactly at one point is called a **tangent.**

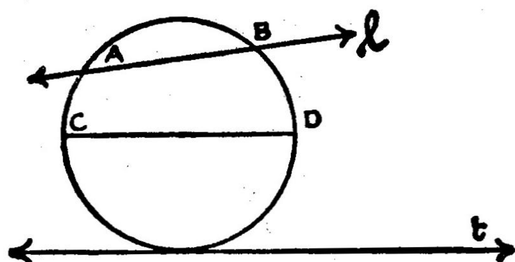


Fig. 5-52.

In Fig. 5-52, *l* is a secant, $\overline{AB}$ is a chord, $\overline{CD}$ is a diameter and t is a tangent. The angle having the centre as the vertex is called the **central angle.**

The central angle helps to measure the arcs.

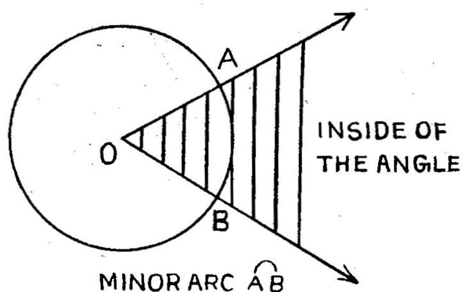


Fig. 5-53.

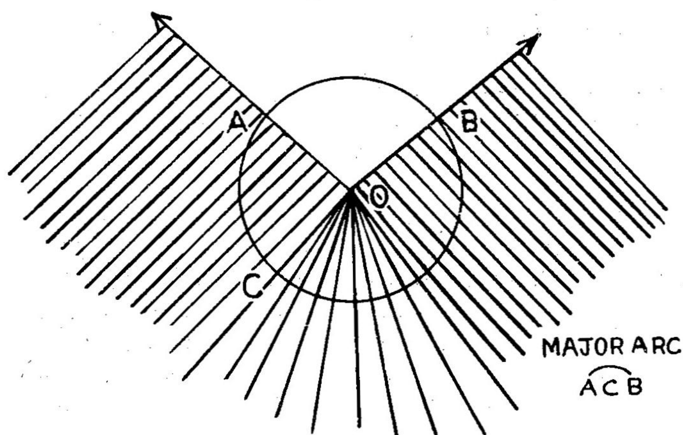


Fig. 5-54.

Arc AB is written as $\widehat{AB}$

m $\widehat{AB}$ represents m $\angle AOB$

In Fig. 5-53, if m $\angle AOB = 60°$,

m $\widehat{AB} = 60°$

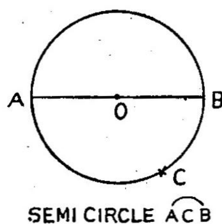


Fig. 5-55.

In Fig 5-54, if m $\angle AOB = 100°$, m $\widehat{ACB} = 360° - 100°$
$= 260°$

In Fig. 5-55, m $\angle$ AOB = 180°; hence m $\angle ACB = 180°$.

Exercise 6—1

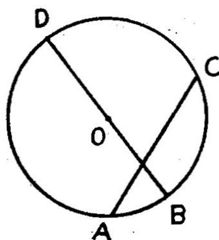


Fig. 5-56.

1. In Fig. 5-56, find (a) minor arcs (b) major arcs (c) semi circles.

2. In Fig. 5-57, $\overline{UX}$ is a diameter.
   (a) If m $\widehat{UV} = 62°$, m $\widehat{WX} = 42°$, find m $\widehat{VW}$.

   (b) m $\widehat{TU} = 72°$, m$\widehat{VW} = 60°$,
   m $\widehat{WX} = 72°$, m $\widehat{XY} = 56°$.
   Find m $\widehat{UV}$ and m $\widehat{YT}$.

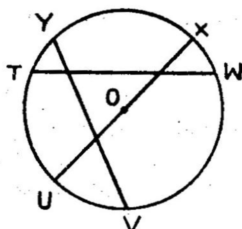


Fig. 5-57.

3. In Fig. 5-58, $\overline{AC}$ and $\overline{BD}$ are diameters.

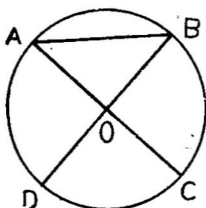


Fig. 5-58.

(a) If m $\widehat{DC}$ = 79° find

m ∠AOD, m ∠ AOB,

m $\widehat{ADC}$ and m $\widehat{BC}$.

(b) If m ∠AOB = 123°, find m ∠BOC, m $\widehat{AB}$,

m $\widehat{BCD}$ and m $\widehat{CAB}$.

6—2. Chords

Draw a circle on a sheet of paper along the edge of a one rupee coin. Cut it off and fold it into two equal parts. The fold is the diameter. Find another diameter. The intersecting point is the centre. Draw any chord. If the paper is folded in such a way that the end points of the chord coincide then we will get the prependicular bisector of the chord. See whether it passes through the centre. What is the distance between the centre of the circle and the mid-point of the chord? Measure it with a match stick.

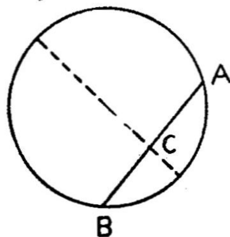


Fig. 5-59.

Draw another chord of the same length and repeat the experiment. From this we can learn that

> If the chords of a circle are congruent, then they are equidistant from the centre.

We can find whether the converse is true.

Converse :

Chords which are equidistant from the centre are congruent.

Plot points in different directions at equal distance from the centre. If the circle is folded along these points we will get different chords. See whether the chords are of equal length. Repeat the experiment. We can understand that the converse is also true.

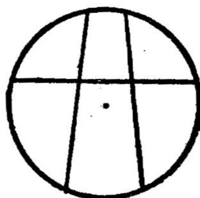


Fig. 5-60.

Chords equidistant from the centre are congruent.

Let us prove the first of the above two theorems mathematically.

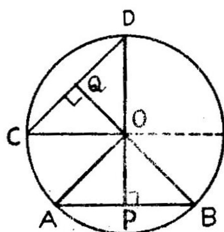


Fig. 5-61.

Given : $\overline{AB} \equiv \overline{CD}$

Proof : Draw $\overline{OP} \perp \overline{AB}$, $\overline{OQ} \perp \overline{CD}$

In $\triangle$ OAB, $\triangle$ OCD,

$\overline{OA} \equiv \overline{OC}$ (radii)

$\overline{OB} \equiv \overline{OD}$ (radii)

$\overline{AB} \equiv \overline{CD}$ (given)

$\therefore \triangle OAB \equiv \triangle OCD$ (SSS)

$\angle OAP \equiv \angle OCQ$

In $\triangle$ OAP, $\triangle$ OCQ,

$\overline{OA} \equiv \overline{OC}$

$\angle OPA \equiv \angle OQC = 90°$

$\angle OAP \equiv \angle OCQ$ (just proved)

$\therefore \triangle OAP \equiv \triangle OCQ$ (SAA)

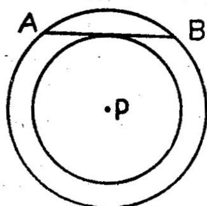
$\therefore \overline{OP} \equiv \overline{OQ}$

Similarly the converse can also be proved.

Exercise 6—2

1. A chord of length 10 cm is at a distance of 12 cm from the centre. What is the radius of the circle ?

2. Find the distance of a chord of length 8 cm from the centre of a circle of radius 6 cm.



3. Two concentric circles of radii 6 cm and 4 cm respectively, have their centre at P. A chord AB of the bigger circle is a tangent to the smaller one. Find the length of AB.

Fig. 5-62.

4. If the distance from the centre of a chord of length 4 cm is twice that of a chord of length 8 cm, find the radius of the circle.

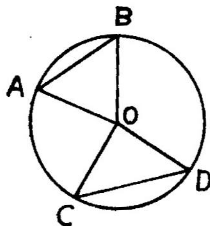
Answers

1. 13 cm 2. $2\sqrt{5}$ cm

6—3. Circles — Arcs and Chords

Take a sheet of paper and cut off a circle. Draw two equal chords. Cut off the circle along chord AB. When the minor segment is placed over the other such that $\overline{AB}$ coincides with $\overline{CD}$, we see that $\overset{\frown}{AB}$ also coincides with $\overset{\frown}{CD}$.

Let us prove it mathematically.



$\overline{AB} = \overline{CD}$ (given)

$\overline{OA} = \overline{OC}$ (radii)

$\overline{OB} = \overline{OD}$ (radii)

$\therefore\ \triangle OAB \equiv \triangle OCD$ [SSS]

Fig. 5-63.

$\therefore\ \angle AOB \equiv \angle COD$

$m\ \overset{\frown}{AB} = m\ \angle AOB\ ;\ \ m\ \overset{\frown}{CD} = m\ \angle COD$

$\therefore$ m $\widehat{AB}$ = m $\widehat{CD}$

$\widehat{AB} \equiv \widehat{CD}$

Hence

> In a circle congruent chords will cut off congruent minor arcs.

Through geometry as well as by paper cutting the converse of this can be found to be true.

> In a given circle congruent arcs determine congruent chords.

You can verify that this will not hold good in the case of circles having different radii. This will hold good only in the case of congruent circles. Hence these theorems can be written as follows :

> In the same or congruent circles, congruent chords determine congruent arcs and congruent arcs determine congruent chords.

Exercise 6—3

1. A, B, C, D, E are 5 points on a circle.

   If m $\widehat{AB}$ = 48°, m $\widehat{AC}$ = 124°, m $\widehat{ABD}$ = 208°

   and m $\widehat{ABE}$ = 312°, find the congruent arcs.

2. $\odot$ O = $\odot$ P. $\widehat{AB}$, $\widehat{CD}$ are chords of $\odot$ O. $\widehat{EF}$, $\widehat{GH}$ are chords of $\odot$ P. If m $\widehat{AB}$ = 118°, m $\widehat{CD}$ = 133°, m $\widehat{EF}$ = 129°, m$\widehat{GH}$ = 118°, find the congruent chords.

3. In Fig, 5-64, $\overline{XW} \cong \overline{YZ}$,
m $\overline{XW}$ = 10t + 5,
m $\overline{YZ}$ = 12t − 21.
Find m $\overline{XW}$.

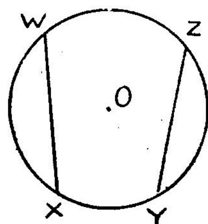


Fig. 5-64.

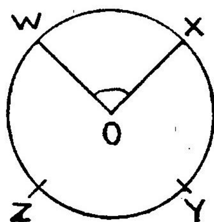
4. A, B, C, D are points on a circle.

(a) AB = 7, BC = 8, CD = 9, DA = 8. Which of the arcs are congruent ?

(b) $\overline{AB} \cong \overline{CB}$. Which of the arcs are congruent ?

(c) m $\widehat{AB}$=150° ; m $\widehat{BC}$ = 80°, m $\widehat{CD}$ = 50°, m $\widehat{DA}$ = 80°. Which of the chords are congruent?

(d) $\widehat{AB} \cong \widehat{CD}$, $\widehat{DA} \cong \widehat{CB}$. Which of the chords are congruent ?



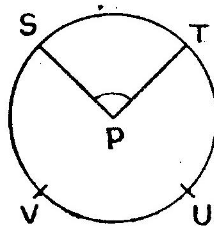


Fig. 5-65.

5. In which of the following cases can you conclude that $\overline{WX}$ = ST ? (refer Fig. 5-65)

(a) $\overline{OW} \cong \overline{PS}$; $\overline{WX} \cong \widehat{TU}$.

(b) OX ≅ $\overline{PS}$; m ∠WOX ≅ m∠SPT

(c) $\overline{OW} \cong \overline{PS}$; m ∠WZX ≅ m∠SVT

6. **In Fig. 5-65, which arcs are congruent?**

$m\,\overset{\frown}{XY} = 85°$; $m\,\overset{\frown}{ST} = 95°$; $m\,\overset{\frown}{XYZ} = 190°$;

$m\,\overset{\frown}{STU} = 180°$; $m\,\overset{\frown}{XZW} = 300°$; $m\,\overset{\frown}{STV} = 240°$

7. **Which arcs are congruent to $\overset{\frown}{AB}$, $\overset{\frown}{BC}$, $\overset{\frown}{DF}$?**

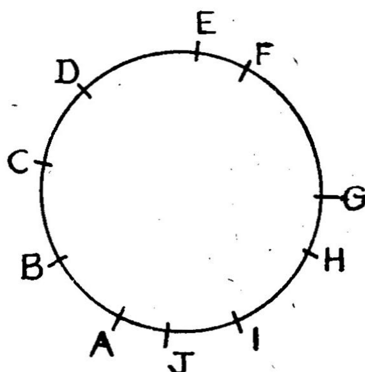


Fig. 5-66.

$m\,\overset{\frown}{AB} = 35°$, $m\,\overset{\frown}{AC} = 75°$, $m\,\overset{\frown}{AD} = 110°$,

$m\,\overset{\frown}{AE} = 160°$, $m\,\overset{\frown}{BF} = 145°$, $m\,\overset{\frown}{BDG} = 215°$,

$m\,\overset{\frown}{BDH} = 240°$, $m\,\overset{\frown}{BDI} - 280°$, $m\,\overset{\frown}{BDJ} = 305°$

8. Given $\overset{\frown}{RY} \cong \overset{\frown}{RL}$ prove that $\triangle$ RLY is isosceles.

9. When $\overset{\frown}{AB} \cong \overset{\frown}{BC} \cong \overset{\frown}{CA}$, what can we say about $\triangle$ ABC ?

10. In Fig. 5-67,

(a) Given $\overset{\frown}{ZXY} = \overset{\frown}{ZYX}$, prove that $\angle \overset{\frown}{ZOX} = \angle \overset{\frown}{ZOY}$

(b) Given $\triangle XOZ \equiv \triangle ZOY$, prove that $X\overset{\frown}{Z} \equiv Y\overset{\frown}{Z}$

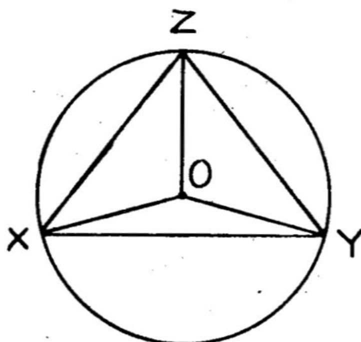


Fig. 5-57.

Answers

1. $\overset{\frown}{AB} \equiv \overset{\frown}{AE}$

6—4. Sectors

When an angle has its vertex at the centre, we call it a **central angle.**

An angle which has its vertex on the circumference is called an inscribed angle.

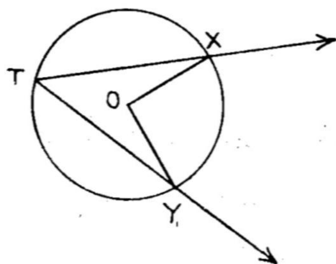


Fig. 5-68.

$\angle XOY$ is a central angle.

$\angle XTY$ is an inscribed angle.

Now let us do some experiments. Cut off a circle from a sheet of paper. When folded along the centre, we get two semi

circles. Take one of the two semicircles and mark a point P on the circle. Join that to end points A, B of the diameter. Fold along the two chords $\overline{PA}$, $\overline{PB}$. See whether ∠APB is a right angle. Take some other points on the arc of the semicircle and repeat the same experiment.

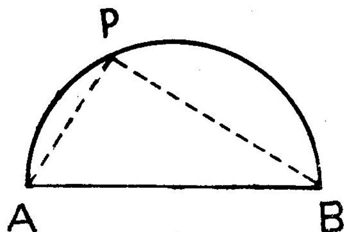


Fig. 5-69.

From this we know that

An angle in a semicircle is a right angle.

Angle in a semicircle implies that the vertex can be at any point on the arc of the semicircle and the rays pass through the end points of the diameter.

Mark any four points on a circle. As seen in the diagram, construct the chords $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DA}$. We get a quadrilateral. Since the vertices are on the circle, it is called a cyclic quadrilateral. Measure ∠ABC, ∠BCD, ∠CDA and ∠DAB,

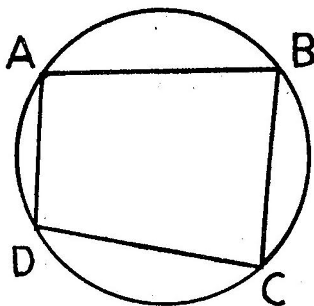


Fig. 5-70.

Find m ∠ABC + m ∠CDA and m∠BCD + m ∠DAB.

You will find that m ∠ABC + m ∠CDA = 180°
m ∠BCD + m ∠DAB = 180°

In a cyclic quadrilateral the opposite angles are supplementary.

You will learn in detail about sectors in the tenth standard. In the next lesson the above two theorems are made use of in constructing certain kinds of quadrilaterals.

Exercise 6—4

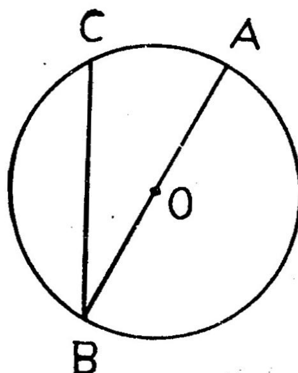


Fig. 5-71.

1. In Fig. 5-71, $\overline{AB}$ is a diameter. Find m ∠ACB.

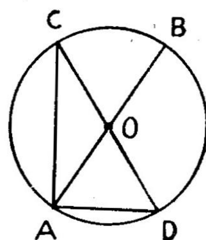


Fig. 5-72.

2. In Fig. 5-72. $\overline{AB}$ ia a, diameter; ∠CAB = ∠BAD.

Prove that m ∠ ABC≡ m ∠ ABD.

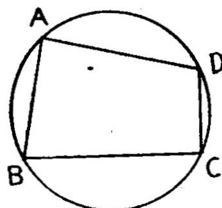


Fig. 5-73.

3. In Fig. 5-73. BD is a diameter; m ∠ADC = 100°.

Find m ∠BAD, m ∠BCD and m ∠ABC.

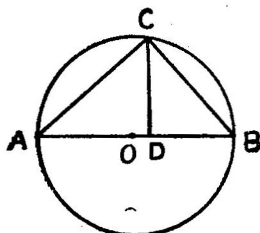


Fig. 5-74.

4. In Fig. 5-74, $\overline{AB}$ is a diameter; CD ⊥ AB.

Show that ∠CAB ≅ ∠ DCB.

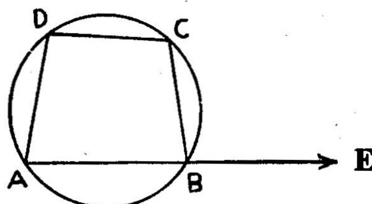


Fig. 5-75.

5. In Fig. 5-75, m ∠CBE = 100°. Find m ∠ADC.

Answers

1. 90°,
2. m ∠BAD = 90° ; m∠BCD = 90° ; m ∠ ABC = 80°
3. m ∠ADC = 100°.

7. Practical Geometry

7 – 1. Trapezium

A quadrilateral having a pair of parallel sides is known as a **trapezium.**

In the previous standard we learnt to construct certain kinds of trapezium.

To construct a trapezium,

(1) We constructed a triangle with the help of 3 measurements.

(2) We then found the fourth vertex with the help of the remaining two measurements.

Now let us see how to construct a trapezium when the four sides are given.

Example :

Draw a trapezium having the following measurements. $\overline{AB} \parallel \overline{DC}$, AB = 8 cm, BC = 4·8 cm, CD = 5 cm, DA = 4 cm.

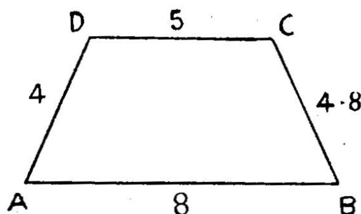


Fig. 5-76.

Draw a rough figure and write down the measurements.

The diagonal $\overline{AC}$ divides the trapezium into two triangles. Do we know the three measurements for any one of the two triangles ? We do not have them. Hence the given measurements are not sufficient to draw △ABC or △ACD. Similarly △ABD or △BCD formed by $\overline{BD}$ cannot also be drawn. Hence let us try to construct a triangle with three of the given measurements.

Since $\overline{AB} \parallel \overline{DC}$, if we draw $\overline{CE} \parallel \overline{AD}$ then we get the parallelogram AECD (refer Fig. 5-77).
In the parallelogram EC = AD = 4 cm, AE = CD = 5 cm.
∴ EB = 3 cm.

Now we have the three measurements for completing the △ ECB.

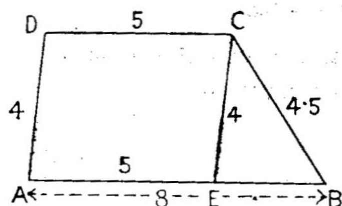


Fig. 5-77.

To draw the trepezium ABCD

(1) Mark the point E on $\overline{AB}$ such that AE = 5 cm.

(2) Draw △ECB.

(3) Taking AD = 4 cm and CD = 5 cm draw arcs and let them intersect at D.

(4) Join CD and DA.

ABCD is the required trapezium.

Exercise 7—1

Draw trapeziums having the following measurements :

1. $\overline{PQ} \parallel \overline{SR}$, PQ = 7·2 cm, QR = 3·8 cm. RS = 4 cm, SP = 3·6 cm.

2. $\overline{AB} \parallel \overline{DC}$, AB = 6 cm, BC = 4 cm, CD = 4·5 cm, DA = 3·5 cm.

3. $\overline{KL} \parallel \overline{NM}$, KL = 4·8 cm, LM = 6 cm, MN = 9 cm, NK = 5 cm.

4. $\overline{AB} \parallel \overline{DC}$, AB = 4 cm, BC = 5·4 cm, CD = 8·5 cm, DA = 5 cm.

7—2. Isosceles trapezium

If the non-parallel sides of a trapezium are equal then it is called an isosceles trapezium.

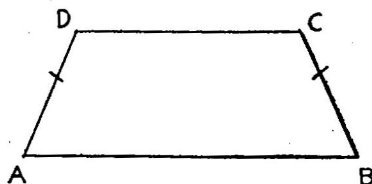


Fig. 5-78.

In ABCD, $\overline{AB} \parallel \overline{DC}$, $\overline{AD} \cong \overline{BC}$.

ABCD is an isosceles trapezium.

Example 1 :

Construct an isosceles trapezium, the parallel sides being 8 cm, 5 cm and the equal sides 4·5 cm.

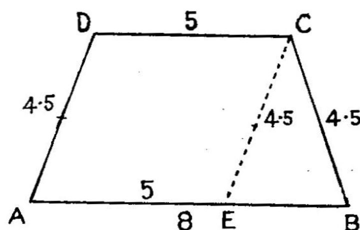


Fig. 5-79.

Have we not studied how to construct a trapezium when the four sides are given ?

Find out m $\angle$A, m $\angle$B, m $\angle$C and m $\angle$D.

Find also the lengths of $\overline{AC}$ and $\overline{BD}$.

You can see that m $\angle$A = m $\angle$B ; m $\angle$C = m $\angle$D and $\overline{AC}$ = $\overline{BD}$.

In an isosceles trapezium the base angles are congruent and the diagonals are congruent.

Example 2 :

In an isosceles trapezium the parallel sides are 7·6 cm and 5 cm; the base angle is 70°. Construct the trapezium.

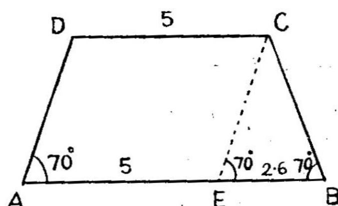


Fig. 5-80.

Fig, 5-80 is the rough figure.

Draw $\overline{CE} \parallel \overline{DA}$

$\angle CEB = \angle DAB$

(1) Draw AB = 7·6 cm in length.

(2) Mark E on $\overline{AB}$ such that AE = 5 cm.

(3) Draw rays such that m $\angle CEB$ = m $\angle CBE$ = 70°. Their point of intersection is C.

(4) Draw $\overrightarrow{AD}$ such that m $\angle BAD$ = 70°.

(5) On $\overrightarrow{AD}$ mark the point D such that CD = 5 cm.

ABCD is the required isosceles trapezium.

Which one will we choose if we get two points for D ?

Exercise 7—2

Draw isosceles trapeziums having the following measurements :

1. The parallel sides are 8 cm, 4·6 cm; the equal sides are 3·5 cm.

2. The parallel sides are 7·6 cm, 3·8 cm; the equal sides are 4 cm.

3. **The parallel sides are 6 cm, 4 cm; the base angles are 60°.**

4. **The parallel sides are 9 cm, 5 cm, the base angles being 75°.**

7—3. Trapezium — Finding the area

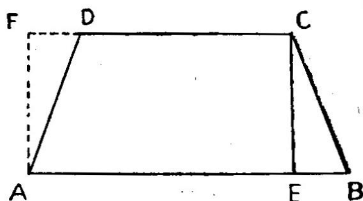


Fig. 5-81.

$\overline{AC}$ divides the trapezium ABCD into two triangles namely △ ABC and △ ACD.

$$\text{Area of } \triangle ABC = \tfrac{1}{2} \times AB \times CE$$

$$\text{Area of } \triangle ACD = \tfrac{1}{2} \times DC \times AF = \tfrac{1}{2} DC \times CE$$

$$\text{Area of trapezium } ABCD = \tfrac{1}{2} AB \times CE + \tfrac{1}{2} DC \times CE$$

$$= \tfrac{1}{2} CE\,(AB + DC)$$

If the parallel sides are a, b and the distance between the parallel sides h, then
the area of the trapezium $= \frac{1}{2} h\,(a + b)$ sq. units.

Example :

The parallel sides of a trapezium are 8 cm, 6 cm. The distance between the parallel sides is 5 cm.

$$\text{Area} = \tfrac{1}{2} \times 5 \times (8 + 6) \text{ cm}^2$$

$$= \tfrac{1}{2} \times 5 \times 14 \text{ cm}^2$$

$$= 35 \text{ cm}^2$$

Exercise 7—3

Find the areas of the trapeziums drawn under Ex. 7—1 and 7-2.

7—4. Construction of a right angled triangle

Recall :

Angle in a semicircle is a right angle.

Example :

Draw the right angled $\triangle$ ABC whose hypotenuse AC = 5 cm and AB = 4 cm.

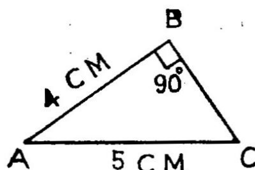


Fig. 5-82.

Since m $\angle B = 90°$, B will be on the semicircle with $\overline{AC}$ as the diameter.

Find the mid-point O of $\overline{AC}$. With O as centre and OA as radius draw the semicircle ABC. Mark B on $\widehat{ABC}$ such that AB = 4 cm.

Note : Can we construct $\triangle$ABC without drawing the semicircle ?

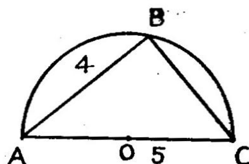


Fig. 5-83.

Exercise 7—4

Construct right angled triangles with the following measurements making use of the semicircle.

1. Hypotenuse 6 cm, one side 4 cm.
2. Hypotenuse 5·4 cm, one side 3·2 cm.
3. Hypotenuse 7 cm, one angle 35°.
4. Hypotenuse 5·8 cm, one angle 48°.

7—5. Construction of Quadrilateral

Recall :

(a) Five measurements are needed to construct a quadrilateral.

Making use of three of the measurements we can complete a triangle and thus find three vertices. With the remaining measurements we can determine the fourth vertex.

(b) Angle in a semicircle is a right angle.

In this lesson let us learn to construct a quadrilateral having 90° as one angle.

Example :

In quadrilateral ABCD, AB = 5 cm, BC = 3·5 cm, CA = 4 cm, AD = 3·6 cm, m ∠D = 90° Construct the quadrilateral.

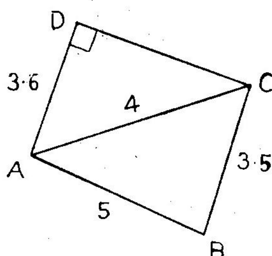


Fig. 5-84.

From the rough figure we know that we have to complete the △ABC and then construct the semicircle with $\overline{AC}$ as diameter. Mark D on $\overset{\frown}{AC}$ such that AD = 3·6 cm.

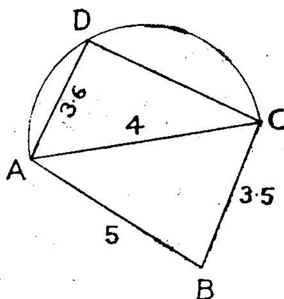


Fig. 5-85.

Exercise 7—5

Construct quadrilateral ABCD with the following measurements.

1. AB = 6 cm, BC = 4 cm, CA = 3·3 cm , AD = 2·8 cm, $m\angle D = 90°$.

2. AB = 6 cm, $m\angle BAC = 40°$, $m\angle ABC = 60°$, CD = 3 cm, $m\angle D = 90°$.

3. AD = 5 cm, $m\angle DAC = 45°$, $m\angle ADC = 60°$, $m\angle B = 90°$, AB = 3·5 cm.

4. $m\angle A = 90°$, $m\angle C = 90°$, BD = 6 cm, DC = 3·5 cm, BA = 3 cm.

7—6. Construction of a cyclic quadrilateral

We learnt that the prependicular bisectors of the sides of a triangle meet at a point known as the circumcentre of the triangle.

Since there is only one circumcircle, we can understand that only one circle can pass through three non-collinear points. Hence to construct a cyclic quadrilateral we have to fix the three vertices of a triangle and draw its circumcircle. The fourth vertex is to be marked on it.

Example :

Construct the cyclic quadrilateral ABCD having AB = 5 cm, BC = 4 cm, CA = 5 cm and AD = 3·5 cm.

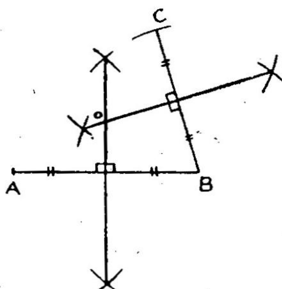


Fig. 5-86.

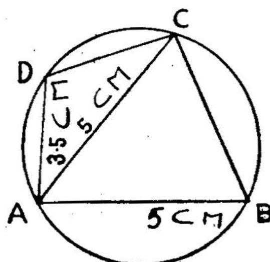


Fig. 5-87.

(1) Construct $\triangle ABC$.

(2) Draw the prependicular bisectors of $\overline{AB}$ and $\overline{BC}$ and let them intersect at O.

(3) Draw a circle having O as centre and $\overline{OA}$ as radius.

(4) Mark a point D on the circle such that AD = 3·5 cm. ABCD is the required cyclic quadrilateral.

Exercise 7 - 6

Construct the cyclic quadrilateral ABCD with the following measurements.

1. AB = 4·8 cm, $m\angle BAC = 48°$, $m\angle BCA = 60°$, $m\angle ABD = 35°$.
2. AB = 6·4 cm, $m\angle BAC = 65°$; AC = 5 cm, AD = 3·6 cm.
3. AB = 7·2 cm, $m\angle ABD = 40°$, $m\angle BAD = 100°$, BC = 4 cm.
4. AC = 7 cm, AD = 4 cm, $m\angle CAD = 48°$, AB = 4·2 cm.

LIFE HISTORY OF MATHEMATICIANS

Descartes :

You must know about Rene Descartes, a gentleman, soldier and mathematician, who had contributed a lot in the field of mathematics. He is considered to be the father of Analytical Geometry that we make use of now.

He was born in a middle class family on 31st March 1596 at La Haye in France. Owing to his delicate health, Charlet, the teacher of La Fleche School, took an instant liking to him and helped him in his studies. During his school days he had the habit of spending his morning hours in deep meditation. At the age of 18, he joined the army. After spending several years as a soldier, he developed a liking for philosophical studies. Later he developed a taste for mathematical research.

His contribution to the development of mathematics is remarkable. He is the founder of the graphs. Graphs have originated from life situations. If we have to locate a house we have to mention whether the street runs from east to west or from south to north and also the door number. In such cases, we come across graphs. With that as basis he realised that two quantities are required to fix a point. This resulted in the birth of Analytical Geometry. He is the man who paved the way for solving equations graphically. The great mathematicians Fermat and Pascal were his contemporaries.

Srinivasa Ramanujam :

One of the greatest of Indian Mathematicians is Srinivasa Ramanujam whose contributions to higher mathematics have been unique and bewildering at the same time. We have known of some of the contributions made by such eminent mathematicians as Aryabhatta, Brahmagupta and Baskara.

Born on December 22, 1882 in a lower middle class family at Erode in Periyar District, Ramanujam had a short life - span of 32 years only. What remarkable 32 years ! He developed a keen interest in mathematics in very early years and solved problems in arithmetic with astonishing ease and quickness. Even as a school boy, he would easily solve university problems. Most problems he would do mentally. He joined the Government Arts College, Kumbakonam but could not go further than the Intermediate (equivalent to 12th standard) due to reasons of poverty.

With the help of his friends, he was able to get an appointment in the office of the Accountant General, Madras, but subsequently joined Madras Port Trust. Wherever he worked, whatever be his job, his primary and sole interest was mathematics. He knocked at the doors of eminent people with his results, but they were rather puzzled at the wide spectrum of his studies. With the assistance of these generous hearted gentlemen, Ramanujam got a Fellowship from the University of Madras for research under Prof. Hardy at the Cambridge University, with whom he had correspondence earlier and who had invited him to England.

He was in England for less than five years from 1914 and did a lot of research. He presented a good number of valuable papers to mathematical journals. He was elected Fellow of the Royal Society, the second Indian to be honoured with this most distinguished award. He returned to India only to die early in 1920.

Ramanujam was a friend of numbers, rather as he used to say, every number was his friend. Once when Prof. Hardy visited him at the hospital, Ramanujam asked what his car number was. Prof. Hardy replied, "1729, not in any way of any speciality about it." Ramanujam replied at once, "How can you say that ? It is the smallest (positive) integer that can be represented as the sum of two cubes in more than one way." Yes, $1729 = 10^3 + 9^3 = 12^3 + 1^3$.

Ramanujam was fond of magic squares. We know that in a magic square the sums of the numbers in the rows, in the columns and in the diagonals are the same. Let us make a magic square with Ramanujam's birth date in the first row.

22	12	18	87
21	84	32	2
92	16	7	24
4	27	82	26

Won't you like to become a great mathematician like Ramanujam ? He was a lonely figure who had to study by himself. You are lucky to have all the opportunities to come up. Be a Ramanujam.

6. GRAPH

1—1. Plotting points on a graph sheet

Let us recall all we have studied in the previous classes about plotting and reading the points on the graph. We learnt that any two mutually perpendicular lines can be taken as the X and Y axes and that every point can be taken as representing an ordered pair with respect to the distances from the X and Y axes. The point (2, — 3) is at a distance of two units on the right side of the X axis and 3 units downwards parallel to the Y axis. It lies in the fourth quadrant.

Exercise 1—1

1. Write down the coordinates of the points plotted in the graph as ordered pairs.

Example : A (2, — 3)

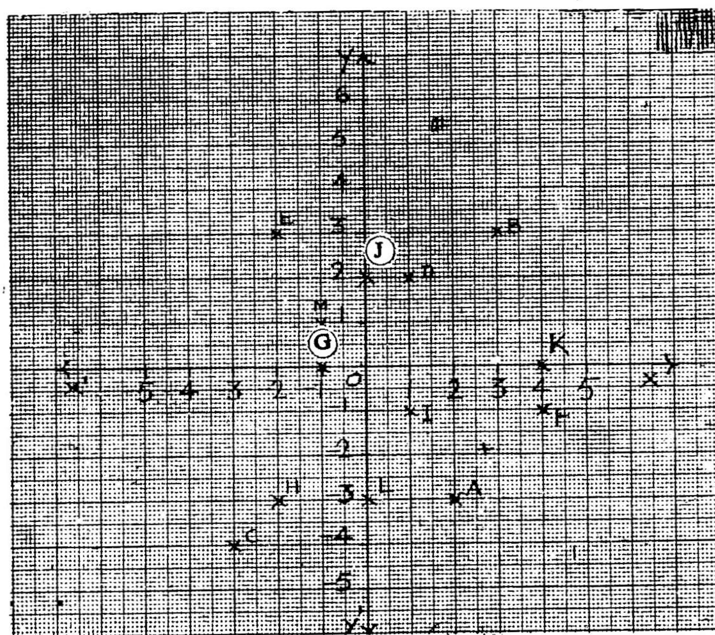


Fig. 6-1.

2. Plot the following points : P (2, 2), Q (— 1, 4), R (3, — 1), S (— 2, — 1), T (3, 1), V (— 2, — 4).

3. By taking the scale 1 cm = 2 units along the X axis and 1 cm = 5 units along the Y axis form a cartesian plane and plot the following points:

(— 4, 20), (8, 10), (— 6, — 15), (6, 15), (— 8, — 10), (4, — 20), (10, — 25), (— 2, 10), (8, 0), (0, — 10), (— 4, 0), (0, 20).

4. What is the co-ordinate of the point of intersection of the axes ?

Answers

1. B (3, 3) ; C (— 3, — 4) ; D (1, 2) ; E (— 2, 3) ; F (4, — 1) ; G (— 1, 0) ; H (— 2, — 3) ; I (1, —1) ; J (0, 2); K (4, 0) ; L (0, — 3) ; M (— 1, 1).

4. (0, 0).

1—2. Graph of y = mx

We have learnt to draw the graph of y = mx in the 8th standard. In Fig. 6-2, the graph of y = 3x is drawn. By looking at the graph carefully, answer the following questions :

1. What is the x coordinate of A ?

2. What is the y coordinate of A ?

3. $\frac{\text{y coordinate of A}}{\text{x coordinate of A}} = \ldots\ldots\ldots$

4. After finding the coordinates of B

find the value of $\frac{\text{y coordinate of B}}{\text{x coordinate of B}}$

5. Find the coordinates of the points C, D, E and find the ratio between the y and x coordinates separately.

6. Does the graph pass through the origin ?

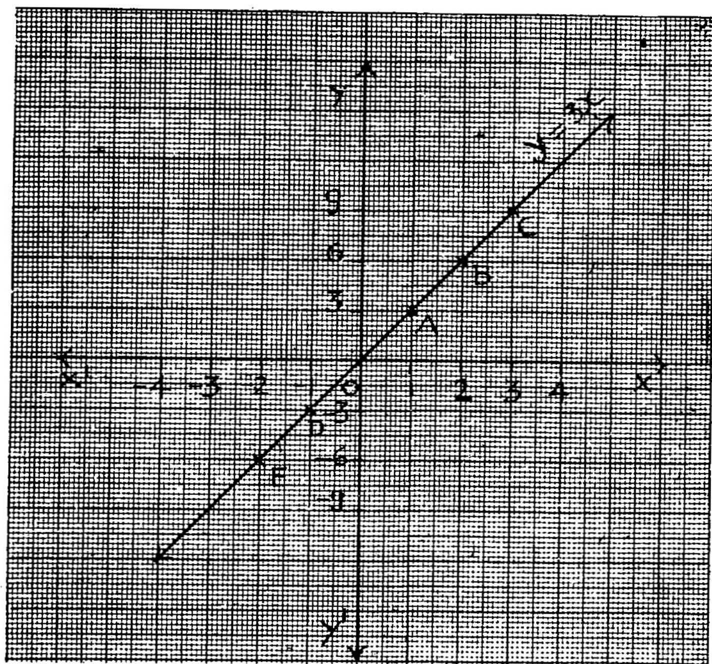


Fig. 6-2.

From this we learn that

(i) The graph of y = 3x is a straight line.

(ii) The origin lies on the line.

(iii) The y co-ordinate of any point on the line is thrice that of the x co-ordinate.

In the graph of y = 3x, 3 is known as the slope or gradient of the line. Can you guess the slope of the line y = 2x ? Draw the graph and verify.

The slope of the line y = mx is m.

Exercise 1—2

1. Draw the graph of y = — 3x.

 (a) Which quadrants does the graph pass through ?

 (b) Mark any ▸ points on the graph and find their co-ordinates.

(c) Find the ratio $\frac{\text{y co-ordinate}}{\text{x co-ordinate}}$

(d) What is the slope of the line ?

(e) Does the y co-ordinate increase or decrease as the x co-ordinate increases ?

2. Find the equations of the lines passing through the origin and having the following slopes :

(a) 4 (b) − 5 (c) 2·5 (d) $\frac{3}{4}$.

3. Write down the ordered pairs of any two points on the line y = 5x.

Find the ratio of the differences between the y co-ordinates and the x co-ordinates.

Similarly find the ordered pairs of 4 more points. Taking two pairs at a time find the ratio of the differences as above. What do you infer from them?

If the co-ordinates of two points on a line are (x_1, y_1), (x_2, y_2), what is the value of $\frac{y_2 - y_1}{x_2 - x_1}$?

Answers

2. (a) y = 4x (b) y = − 5x
(c) y = 2·5x (d) y = $\frac{3}{4}$x.

1—3. Graph of y = mx + c

Taking the scale 1 cm = 1 unit along the X axis and taking 1 cm = 4 units along the Y axis, draw the graph of y = 3x.

Let us try to draw the graph of y = 3x + 4.

x	0	2	4
y (= 3x + 4)	4	10	16

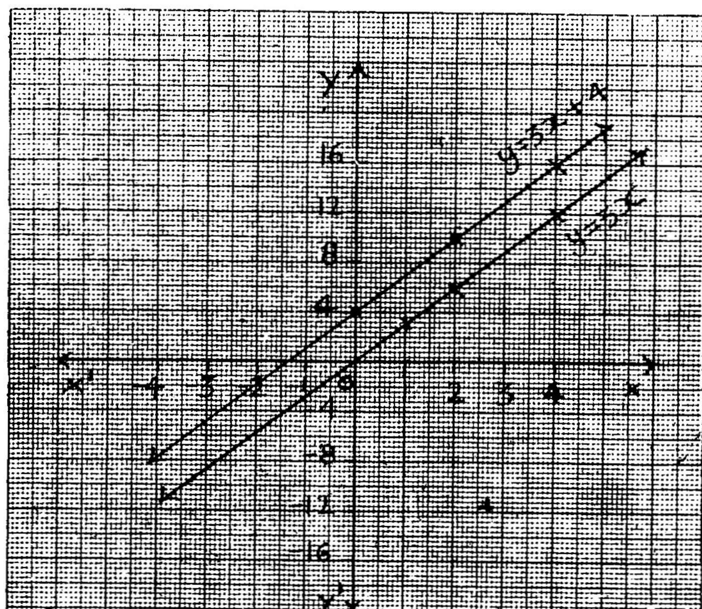


Fig. 6-3.

(a) 1. Complete the following table :

x	0	1	2	3	4
y	4		10		16
y − 4	0				

2. Can you find the suitable values of y for the consecutive values of x ?

3. Can you infer the value of y when x = 5 ?

4. Find the differences between the values of y and x.

5. Compare the graph of $y = 3x$ and $y = 3x + 4$. What can we infer ?

6. Find the ratio $\frac{y-4}{x}$ in each case.

(b) From the above you know that

1. The line $y = 3x + 4$ is parallel to the line $y = 3x$.

2. The two lines have the same slope. Their slope is 3.

3. The graph of $y = 3x + 4$ cuts the y axis at $y = 4$ that is, at (0, 4).

4. In the equation $y = 3x + 4$, 3 is the slope and 4 is the intercept which the line makes on the Y axis. It is known as Y intercept.

(c) To draw the graph of $y = 3x + 4$,

1. Mark the point (0, 4) on the Y axis.

2. From that point move 1 unit to the right in the direction parallel to the X axis and mark the point after moving 3 units upward in the direction parallel to the Y axis.

3. Join the two points and produce the line in both directions.

(d) Draw the graph of $y = 3x - 5$ and find its slope and Y intercept.

Exercise 1—3

1. Find the slopes and Y intercepts of the following lines :

(a) $y = 2x + 5$ (b) $y = 4x - 3$

(c) $y = 3x - 4$ (d) $y = 8 - 5x$

2. Draw the graph of the above equations.

3. Find the ordered pairs of any four paints on the line $y = 5x - 2$. Choosing any two points at a time, find the ratio of the difference between the second terms and the first terms. What can we infer?

4. If (x_1, y_1) and (x_2, y_2) are two points on a line, what will $\frac{y_2 - y_1}{x_2 - x_1}$ represent?

Answers

1. (a) Slope = 2, Y intercept = 5,
(b) Slope = 4 , Y intercept = -3
(c) Slope = 3, Y intercept = -4
(d) Slope = -5, Y intercept = 8.

1—4. Reading the graph

Often we can see line diagrams or broken line diagrams in the newspapers to illustrate price index, budget figures, production of commodities etc. We must have a knowledge of these diagrams. Let us see what the word 'line diagram' means.

(a) Positive and negative slopes

In $y = mx + c$ we know that m is the slope and c the y intercept. Later we learnt that $y = mx + c$ is an increasing function when m is positive and a decreasing one when m is negative. That is, if y increases, when there is an increase in x, then m is positive and the line is said to have a positive slope. On the other hand if y decreases when there is an increase in x, then m is negative and the line is said to have a negative slope. Hence the graph of a line will represent an increasing function if on moving from left to right it moves from the bottom to the top. Here m is positive. Tha graph of a line will represent a decreasing function if on moving from left to right it comes down from the top to the bottom. Here m is negative.

In the following graphs identify the increasing and decreasing fuuctions :

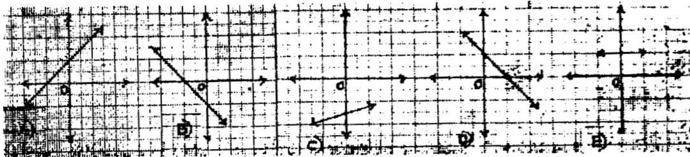


Fig. 6-4.

In Fig. 6-4 (E), the value of y neither increases nor decreases. It is a constant function. What do you know about it ?

(b) Finding the y intercept of a line

We learnt that c is the intercept which the line $y = mx + c$ makes with the Y axis. To find c it is enough if we find the y co-ordinate of the point where the line meets the Y axis.

Find the y intercepts in the following graphs :

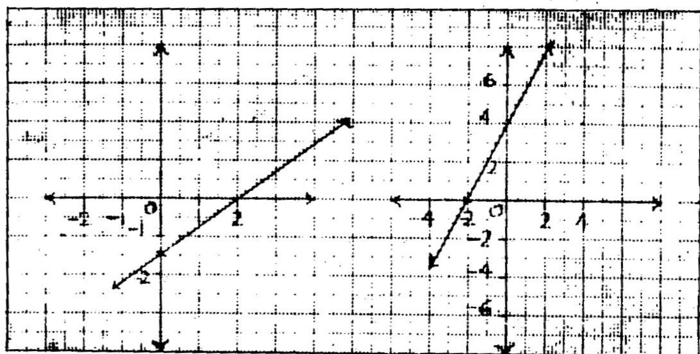


Fig. 6-5.

(c) Finding the slope of a line

Already we know that if the points (x_1, y_1) and (x_2, y_2) lie on the graph of $y = mx + c$, then $m = \frac{y_2 - y_1}{x_2 - x_1}$

If the points (5, 2), (3, 4) lie on the graph of a line then the slope of the line $m = \frac{4 - 2}{3 - 5} = -1.$

If (0, 3), (1, 5) are two points on a line, its slope m is $\frac{5 - 3}{1 - 0} = 2.$

When $x = 0$, the value of y is the intercept which the line makes with the Y axis.

When the x co-ordinates are 0, 1, the slope of a line is the difference between the y co-ordinates.

If (0, $-$ 2) and (1, 3) are two points on another line you can see the slope of the line to be $3 - (-2) = 5$.

(d) Finding the equation of the line

So far we learnt the methods of finding the slope m as well as the y intercept and c or a line from a given graph. We also know that if the slope of a line is m and the y intercept is c then its equation is $y = mx + c$.

Find the equations of the following graphs:

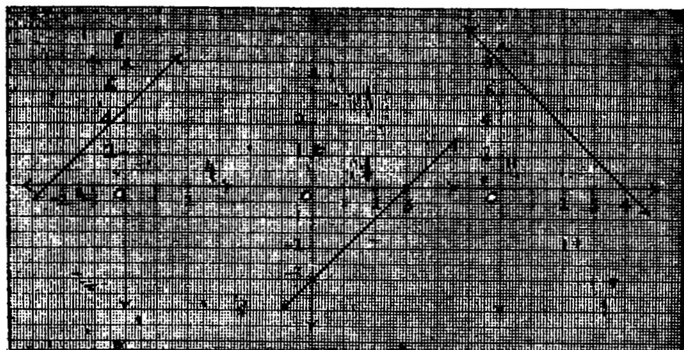


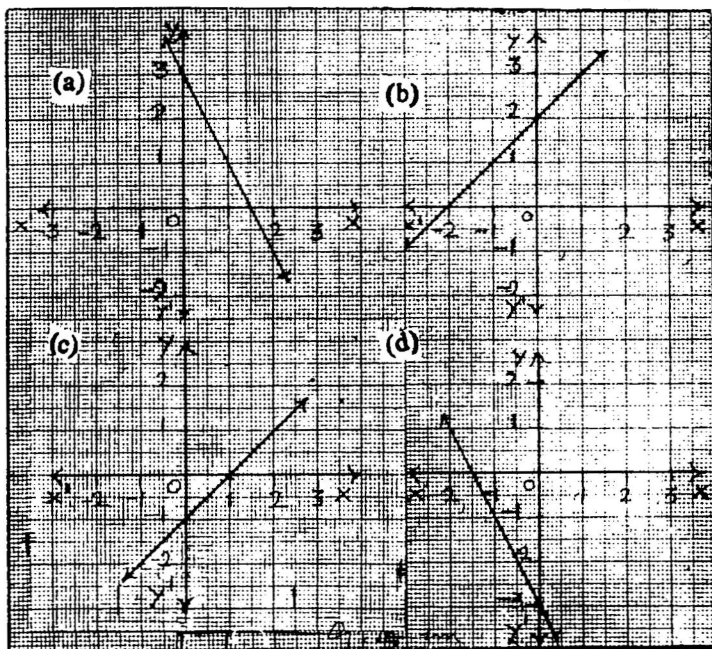
Fig. 6-6.

Answers

$y = 2x + 5$; $y = x - 3$; $y = 2x + 8$

Exercise 1—4

1. Find the equations of the following graphs :



2. The following points are on the respective lines given against them. Find the missing number of the ordered pairs.

line (a) (1, ...), (2, ...), (..., 5), (..., 2)

line (b) (2, ...), (..., 7), (..., — 2), (..., 0)

line (c) (..., 3), (—3, ...), (3, ...) (..., —3)

line (d) (..., 1), (2, ...), (..., —9), (—3, ...)

3. If a point on the line d is (a, a) find the value of 'a'.

Answers

(a) $y = 3 - 2x$ (b) $y = x + 2$

(c) $y = x - 1$ (d) $y = -2x - 3$

1—5. Making use of the graph $y = mx + c$

(a) A man buys a gramaphone record player for Rs. 500 and some records at the rate of Rs. 50 per record. This can be represented in an equation as $y = 500 + 50x$. The total cost depends upon the number of records.

(i) What does 500 represent?

(ii) What does 50 represent?

The cost of the record player is Rs. 500. It is the y intercept of the line. The cost of a single record is Rs. 50. It is the slope of the line. Draw the graph of the equation and find the total cost of a record player and 10 records.

(b) The cost of a record player with 6 records is Rs. 670 and the cost of the same record player with 10 records is Rs. 850. Shall we find the cost of the record player?

(i) Plot the points (6, 670) and (10, 850) on the graph sheet.

(ii) Join the points and produce the line.

(iii) At which point does the line cut the Y axis? What does it represent?

(iv) How can we find the cost of a record from the graph?

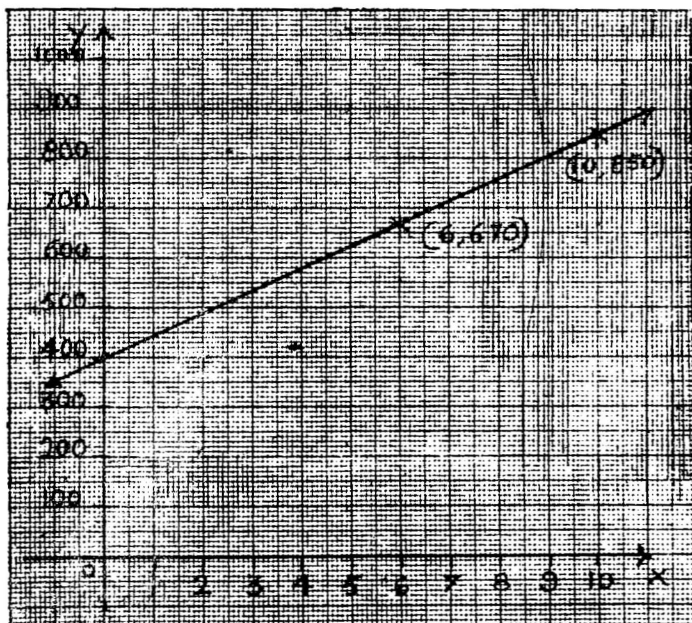


Fig. 6-8.

We know that the slope of the line represents the cost of a single record.

$$\text{Slope } m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{850 - 670}{10 - 6} = 45.$$

Exercise 1—5

1. A sum invested at a particular rate of simple interest amounts to Rs. 1300 in 3 years and Rs. 1500 in 5 years. Find the principal and the interest per year.

2. A person bought a tape recorder along with 10 tapes for Rs. 1600 and his friend bought the same kind of tape

recorder with 20 tapes for Rs. 2100. Find the cost of the tape recorder and the cost of a single tape.

3. A person travelling at a uniform speed was 120 km away from a particular place after a journey of 2 hours and 40 km away after 4 hours. Find how far he was at the beginning of the journey. Find the speed.

Answers

1. Rs. 1000, Rs. 100 2. Rs. 1100, Rs. 50
3. 40 km/hour, 200 km.

2—1. Graph of $ax + by + c = 0$

Let us try to draw the graph of the equation $2x—3y—6=0$. It is an equation in x and y and the power is 1. Hence we can infer that it represents a straight line. We know that we can draw a straight line if we know two of the points on it.

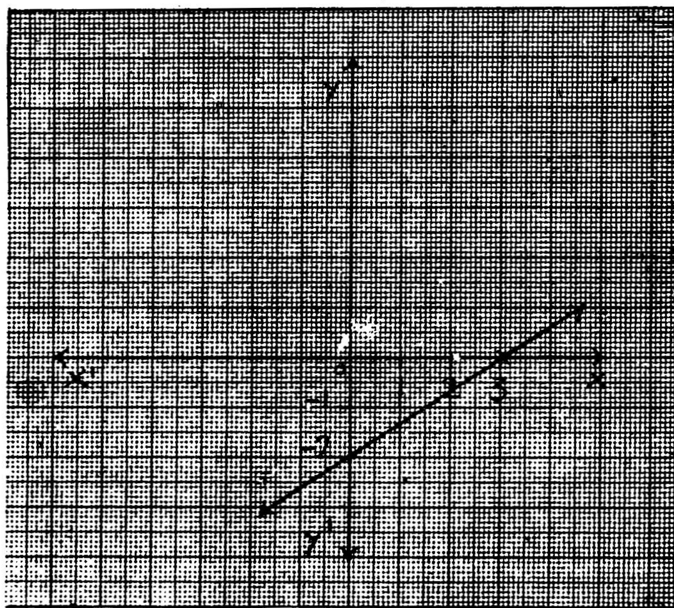


Fig. 6-9.

Let us change the equation $2x - 3y - 6 = 0$ to the form $y = mx + c$.

$$-3y = 6 - 2x$$
$$3y = 2x - 6$$
$$y = \frac{2}{3}x - 2$$

From this we know the slope of the line to be $\frac{2}{3}$; the y intercept is -2.

When $x = 0$, $y = -2$. Hence $(0, -2)$ is a point on the line. When $x = 3$, $y = 0$. Hence $(3, 0)$ is another point on the line.

By plotting these two points and joining them, we get the graph of $y = \frac{2}{3}x - 2$ i.e. $2x - 3y - 6 = 0$.

In this figure we find the x intercept to be 3 and the y intercept -2.

An easy way of drawing lines of the form $ax + by + c = 0$ is to find the x and y intercepts and join them.

What is the x intercept of the equation $3x + 4y + 6 = 0$? What is its y intercept? What is its slope?

Exercise 2—1

1. Draw the graphs of the following equations. Find their slopes.

   (a) $2x + 3y = 6$ (b) $3x - 2y - 12 = 0$

   (c) $6x + 5y + 15 = 0$.

2. What is the slope of $ax + by + c = 0$? Find the x and y intercepts. Examine whether there is any relationship among the x intercept, y intercept and the slope.

Answers

1. (a) Slope $= -\frac{2}{3}$ (b) Slope $= \frac{3}{2}$ (c) Slope $= -\frac{6}{5}$

2. Slope $= \frac{-a}{b}$, y intercept $= \frac{-c}{b}$,

   x intercept $= \frac{-c}{a}$

2—2. Graph of the form $\frac{x}{a} + \frac{y}{b} = 1$

We learnt to find out the x and y intercepts of the equation $ax + by + c = 0$.

The x intercept of $2x - 3y - 6 = 0$ is 3 and the y intercept is -2.

$$2x - 3y - 6 = 0$$

$$\therefore \; 2x - 3y = 6$$

Dividing both sides by 6,

$$\frac{2x}{6} - \frac{3y}{6} = 1 \text{ or } \frac{x}{3} + \frac{y}{-2} = 1.$$

Look at the x and y intercepts of this form.

If $\frac{x}{5} + \frac{y}{2} = 1$, what is the x intercept ?

what is the y intercept ?

If $\frac{x}{-2} + \frac{y}{3} = 1$, what is the x intercept ?

what is the y intercept ?

Change the equations of the graphs of the previous exerçise to the form $\frac{x}{a} + \frac{y}{b} = 1$.

x intercept of the line of the form $\frac{x}{a} + \frac{y}{b} = 1$ is 'a'; y intercept is 'b'. Verify this by substituting $x = 0$ and $y = 0$ respectively in the given equation.

We have now learnt an alternate method to draw the graph of $ax + by + c = 0$ and to find the x and y intercepts.

If $3x - 5y - 12 = 0$, then $3x - 5y = 12$

$$\text{or } \frac{3x}{12} - \frac{5y}{12} = 1 \text{ or } \frac{x}{4} - \frac{y}{2{\cdot}4} = 1$$

The x intercept is 4 and the y intercept 2·4.

Have we not learnt that when the x and y intercepts are known, slope $m = -\frac{\text{y intercept}}{\text{x intercept}}$?

In the above problem find 'm' and verify.

Exercise 2—2

1. Change the following equations to the form $\frac{x}{a} + \frac{y}{b} = 1.$

	Equation	x intercept	y intercept	Equation in the form $\frac{x}{a} + \frac{y}{b} = 1$
(a)	$2x + 5y = 15$			
(b)	$4x - y = 8$			
(c)	$2x + 3y = 9$			
(d)	$x + y = 4$			

2. Express the equations of the following graphs in intercept form Then change them to the form $ax + by + c = 0$

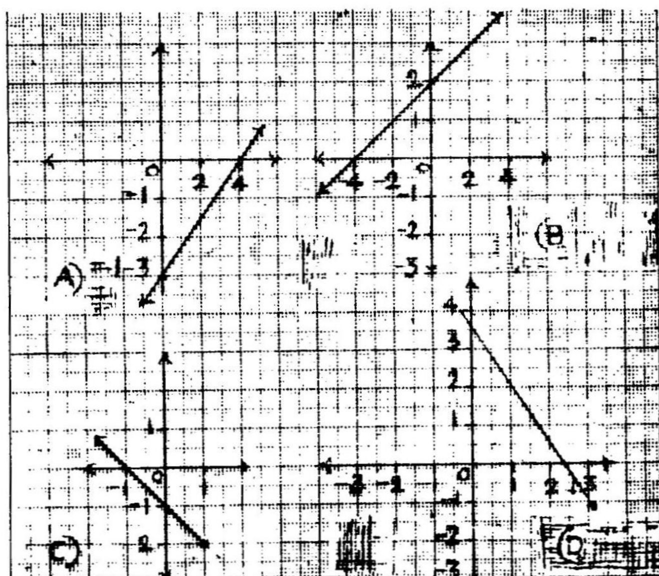


Fig. 6-10.

3. Find the intercept of the following equations and draw their graphs :

(a) $3x - 6y = 12$ (b) $2x + 3y - 9 = 0$

(c) $4y - 3x + 6 = 0$ (d) $\frac{x}{2} + \frac{y}{4} = 2.$

Answers

1\. (a) $\frac{x}{15/2} + \frac{y}{15/5} = 1$ (b)] $\frac{x}{8/4} + \frac{y}{-8} = 1$

(c) $\frac{x}{9/2} + \frac{y}{9/3} = 1$ (d) $\frac{x}{4} + \frac{y}{4} = 1$

2\. (a) $\frac{x}{4} + \frac{y}{-3} = 1$; $3x - 4y - 12 = 0$

(b) $\frac{x}{-4} + \frac{y}{2} = 1$; $x - 2y + 4 = 0$

(c) $\frac{x}{-1} + \frac{y}{-1} = 1$; $x + y + 1 = 0$

(d) $\frac{x}{2{\cdot}4} + \frac{y}{3{\cdot}6} = 1$; $3x + 2y - 7{\cdot}2 = 0$

3\. (a) 4, -2 (b) $\frac{9}{2}$, 3 (c) 2, $-\frac{3}{2}$ (d) 4, 8

3—1. Solving linear equations (a)

So far we learnt to draw the graph when the equation of the line is given and to read the equation when the graph is given. Now let us learn how to solve two linear equations graphically.

We know that non-parallel lines meet at a single point and that that point alone is the common point of the two lines.

Hence to solve two linear equations :

(1) Draw their graphs

(2) Find the co-ordinates of the intersecting point

The co-ordinates are represented as an ordered pair and this ordered pair is the solution set of the two equations.

Example :

$2x - y = 3$ and $x + 2y = 4.$

We can see that the x intercept of $2x - 3y = 3$ is 1·5 and the y intercept is -3.

The x intercept of $x + 2y = 4$ is 4 and the y intercept is 2.

Draw the graph of these and find the point of intersection. The point of intersection is (2, 1).

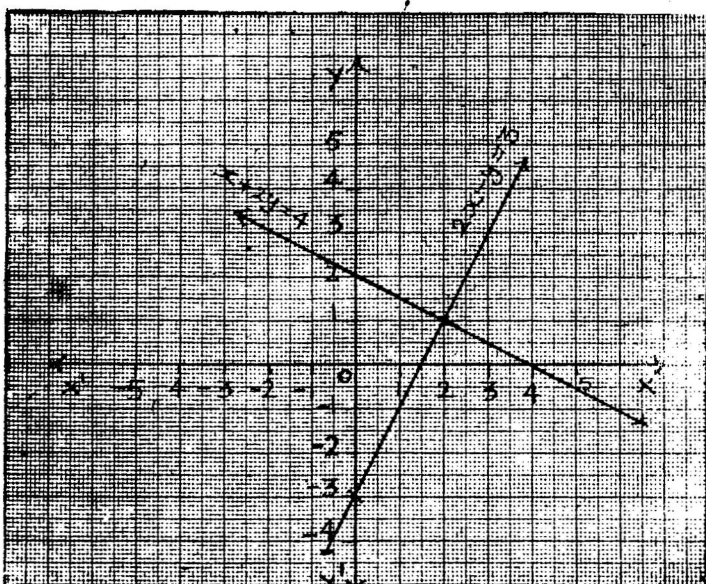


Fig. 6-11.

Hence the solution set of $2x - y = 3$ and $x + 2y = 4$ is $\{ (2, 1) \}$.

Exercise 3—1

1. Find the solution set of the following equations graphically.

(a) $2x - y = 8$ (b) $x - 2y = -1$ (c) $3x - y = 2$

$x + 2y = 5$ $\frac{x}{4} + \frac{y}{3} = 1$ $2x + y = 8$

Answers

(a) $\{ (\frac{21}{5}, \frac{2}{5}) \}$ (b) $\{ (2, \frac{3}{2}) \}$ (c) $\{ (2, 4) \}$

3—2. Solving linear equations (b)

We know how to solve linear equations graphically when the solution set lies in the I quadrant. Now we shall learn to solve equation whose solution set is in any one of the quadrants.

Example :

$x + y = -3$ and $2x - 3y = 4$.

The x intercept of $x + y = -3$ is -3; the y intercept is -3

The x intercept of $2x - 3y = 4$ is 2; the y intercep is -1.3.

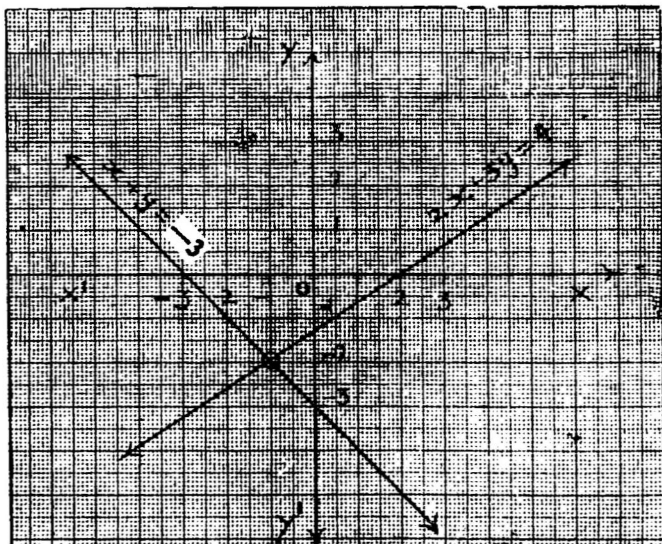


Fig. 6-12.

On drawing the graph, we can find out the point of intersection to be $(-1, -2)$.

The solution set is $\{ (-1, -2) \}$.

Exercise 3—2

1. Find the solution set graphically :

(a) $\frac{x}{2} - \frac{y}{15} = 1$ (b) $x + 2y = 1$ (c) $2x + 3y = -1$

$2x - y = -3$ $\quad 2x + y = 5$ $\quad x + y = -1$

2. What are the x and y intercepts of the graph $2x + 3y = 0$? What do you infer ?

3. What are the slopes of the equations given in exercises 3 - 1 and 3 - 2? What is there in common ?

Answers

1. (a) $\left\{ \left(\frac{4}{9}, \frac{35}{9} \right) \right\}$

(b) $\{ (3, -1) \}$ (c) $\{ (-2, 1) \}$

2. x intercept = 0, y intercept = 0.

3—3. Linear equations (c)

So far the slope of one equation was positive and the other was negative. Now let us learn to solve equations having either positive slopes or negative slopes.

(1) What are the slopes of $2x + 3y = 6$ and $4x + 6y = 18$?

Draw their graphs and find their point of intersection. If the slopes are equal, the lines are parallel. They do not intersect at real points. These equations are called inconsistent equations.

What is the solution set of such equations ? Examine whether it is a null set.

The slopes of inconsistent equations are equal. What can we infer about the slopes of intersecting lines having equal slopes?

$y = 7x + 5$ and $y = 3x + 1$ are two equations of intersecting line. Are they inconsistent ? Why ?

Exercise 3 – 3 (1)

Which of the following pairs of equation are inconsistent ?

(a) $2x - 3y = 7$; $3x + 2y = 5$

(b) $2x - 3y = 7$; $2x - 3y = 3$

(c) $3x + 2y = 5$; $2x - 3y = 7$

(d) $3x + 2y = 5$; $9x + 6y = 20$

(e) $7x - 5y = 12$; $14x - 10y = 18$

Answers

(b), (d), (e) are inconsistent equations,

2. $y = 2x - 1$ and $2y = 3x + 1$

When $y = 2x - 1$, what is the slope? What is y intercept ?

When $2y = 3x + 1$, $y = \frac{1}{2}x + \frac{1}{2}$. Its slope is $\frac{3}{2}$ and y intercept is $\frac{1}{2}$.

Now let us draw their graphs.

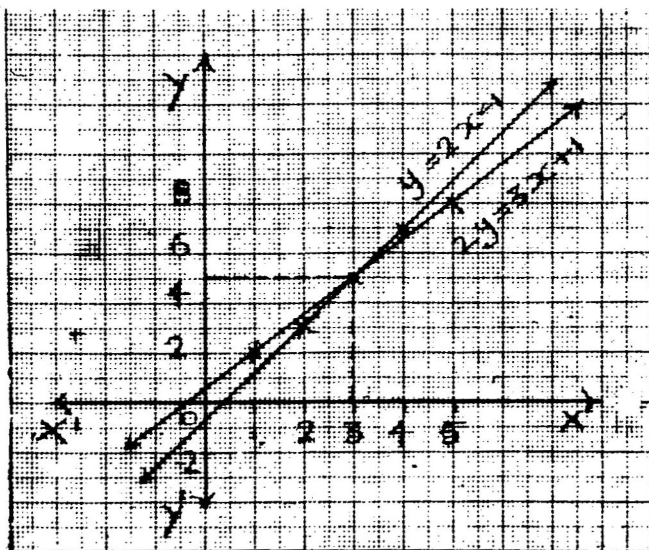


Fig. 6-13.

Are these increasing or decreasing functions ?

(3) The graphs of two functions are given below. What kind of functions are they? What kind of slopes do they have? Find their equations. With the help of the figure find their solution set.

Since the x intercept of the 1st line is 2 and y intercept 5, its equation is $\frac{x}{2} + \frac{y}{5} = 1$ or $5x + 2y = 10$.

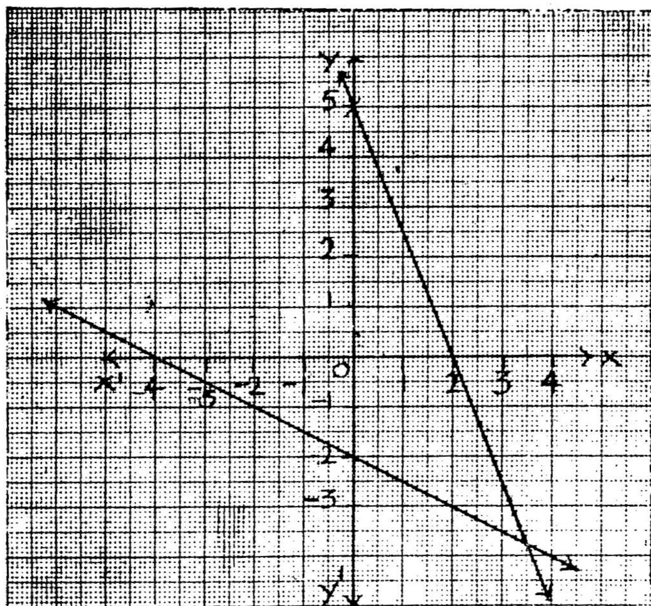


Fig. 6-14.

The x intercept of the second line is — 4 and the y intercept — 2. Its equation is $\frac{x}{-4} + \frac{y}{-2} = 1$ or $x + 2y = -4$.

Exercise 3—3 (2)

1. Find the solution sets of the following :

(a) $y = 3x$; $x - y = 4$ (b) $x = 4$; $x + y = 1$

(c) $x - y = 1$; $y = 3$.

2. Find the equations and solution sets of the following graphs :

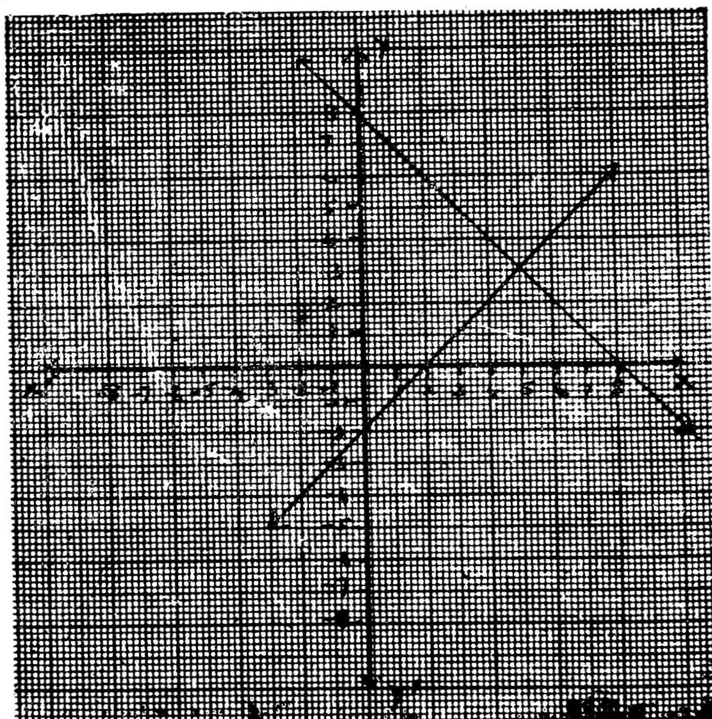


Fig. 6-15 (a).

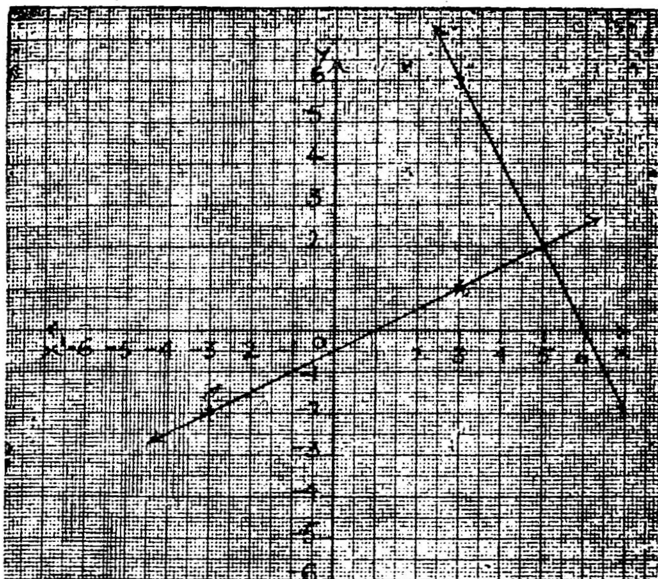


Fig. 6-15 (b).

Answers

2. (a) $x + y = 8$; $x - y = 2$; $x = 5$; $y = 3$

   (b) $2x + y = 12$; $x - 2y = 1$: $x = 5$; $y = 2$

3—4. Linear equations (Uses)

In this lesson we shall learn to form equations and then solve them.

At first let us recall all that we have studied about mathematical sentences.

If y is twice that of x, then $y = 2x$

If y is greater than x by 3 then $y = x + 3$

If y is less than x by 5 then $y = x - 5$

If y exceeds two times x by 4 then $y = 2x + 4$

Like this, the statement two times x and three times y sum up to 9 is written in mathematical sentence as $2x + 3y = 9$.

Some statements and their corresponding equations are given below. Study them carefully.

	Statement	Equation
1.	Three times y is equal to two times x	$3y = 2x$
2.	Two times y exceeds five times x by 3	$2y = 5x + 3$
3.	3 times x exceeds 2 times y by 7	$3x - 2y = 7$
4.	5 times x is less than 2 times y by 3	$5x = 2y - 3$

Let us now see another kind of sentence.

Muthu buys 18 pencils and 7 pens for Rs. 20.

If we take the cost of 1 pencil as Rs. x and 1 pen as Rs. y then we can form the mathematical sentence $18x + 7y = 20$.

The cost of four 80 pages note-books is less than the cost of 3 one quire note-books by Rs. 2-80.

If we take the cost of one 80 pages note-book as x paise and the cost of a 1 quire note-book as y paise, then the mathematical sentence is

$$4x = 3y - 280 \text{ or}$$
$$3y = 4x + 280.$$

Observe that both sides of the sentence are in the same unit namely paise.

Exercise 3—4

Frame mathematical sentences:

1. y is 5 times x
2. y exceeds x by 7
3. y is $\frac{2}{3}$ of x
4. y is less than two times x by 9.
5. $x > y$. Their difference is 7.
6. $x < y$. Difference between them is 3.
7. Two times x is less than 3 times y by 5
8. 5 times x exceeds 2 times y by 7
9. Subtracting 5 times x from 3 times y we get -6
10. 5 times 3 added to y is equal to 9 added to x.

Answers

1. $y = 5x$	2. $y = x + 7$	3 $y = \frac{2}{3}x$
4. $y = 2x - 9$	5. $x - y = 7$	6. $y - x = 3$
7. $3y - 2x = 5$	8. $5x - 2y = 7$	9. $3y - 5x = -$
10. $5(y + 3) = x + 9$		

4—1. Inequalities — Revision: $x > a$, $x < a$ $(x \in R)$

In the VIII Standard we have learnt to represent graphically an inequality in a single variable. Let us revise the same.

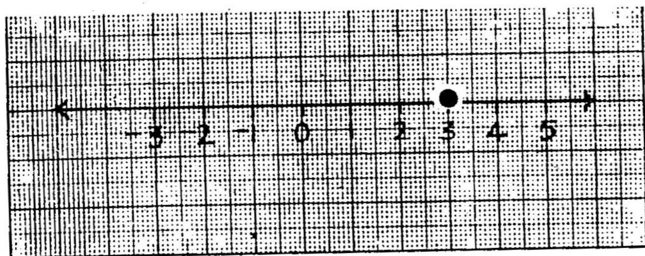


Fig. 6 - 16.

The equation x = 3 can be shown in the number ray by a big dot at 3.

When $x > 3$ $(x \in Z)$, the solution set of x is $\{4, 5, 6, ...\}$. If these points are marked on the number ray we will get the graph of $x > 3$.

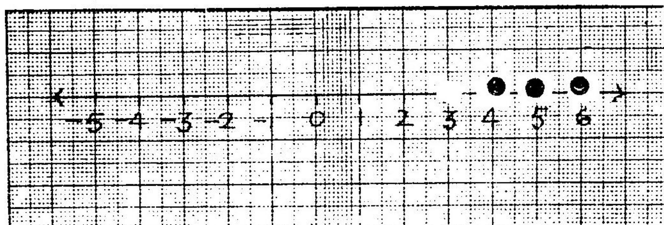


Fig. 6 - 17.

When $x > 3$ $(x \in Q)$, the solution set includes all rational numbers greater than 3. This can be represented graphically by drawing a hollow circle at 3 on the number ray followed by bold broken lines on the right.

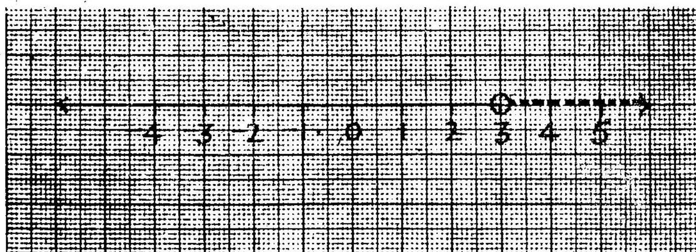


Fig. 6 - 18.

What do the gaps represent ?

When the solution set lies in the real number system there will not be any gap in the number ray. Hence, the graph of $x > 3$, $x \in R$ will be as shown in Fig. 6—19.

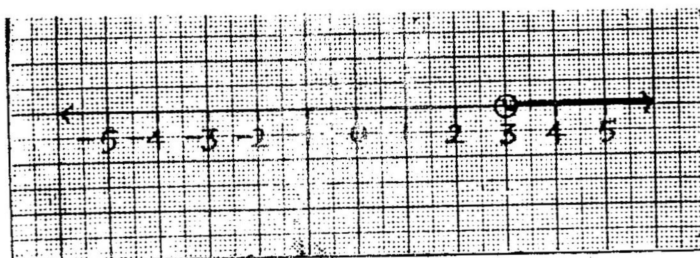


Fig 6 - 19.

The graph of $x \geqslant 3$, $x \in R$, is the union of the graphs $x > 3$ and $x = 3$.

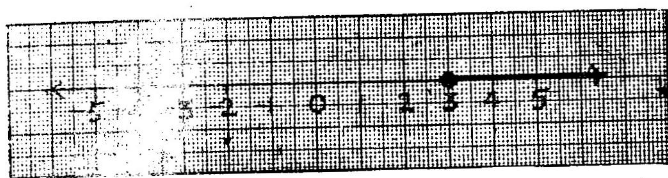


Fig. 6 - 20.

The one given below represents the graph of $x \leqslant 3$, $x \in R$.

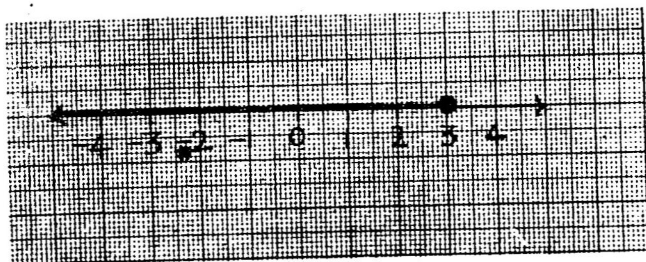


Fig. 6 - 21.

Exercise 4—1

1. Find the solution sets of the following eqations and the inequalities graphically by taking the solution sets to be in (1) the set of real numbers (2) the set of rational numbers (3) the set of integers.

(a) $x = 4$ (b) $x > 4$ (c) $x > -4$

(d) $x < 4$ (e) $x < -4$ (f) $x \geqslant 4$

(g) $x \leqslant 4$ (h) $x \geqslant -4$ (i) $x \leqslant -4$.

2. Find the inequalities represented by the following graphs and also the set in which the solution set lies.

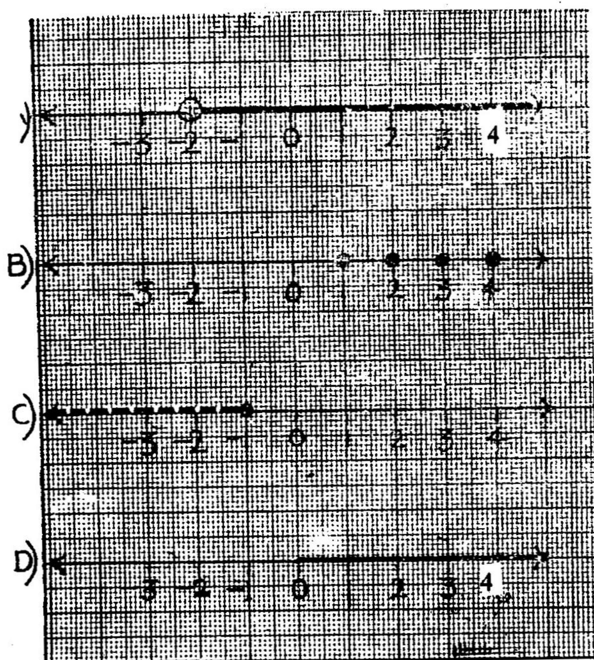


Fig. 6 - 22.

Answers

(a) $x > -2$, $x \in R$ (b) $1 \leqslant x$, $x \in Z$

(c) $x \leqslant -1$, $x \in R$ (d) $x \geqslant 0$, $x \in R$.

4—2. Inequalities : $a \geqslant x \leqslant b$

In the last lesson we saw the graphs of inequalities of the form $x > a$ and $x \leqslant b$. Let us now learn about the inequalities which are the combinations of the above two types.

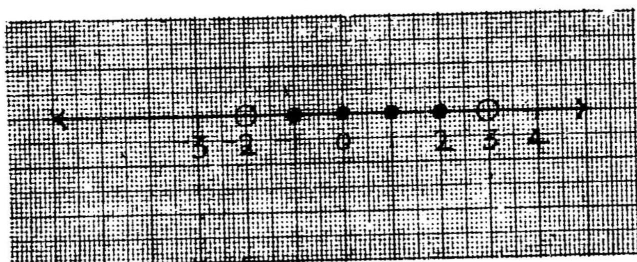


Fig. 6 - 23.

In the above figure the solution set is $\{-1, 0, 1, 2\}$ Since the limits of this are -1, 2, the inequality representing the graph is $-2 < x < 3, x \in Z$.

The graph of $-1 \leqslant x \leqslant 2, x \in Z$ is given below.

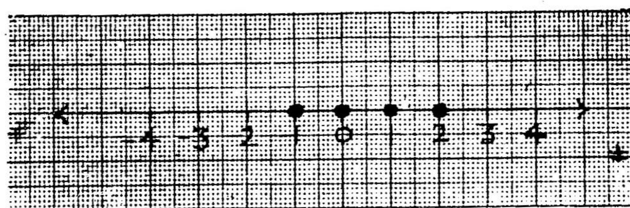


Fig. 6 - 24.

Note the difference between the above two inequality graphs (Fig. 6-23 and Fig. 6-24).

The one given below is the graph of $-2 < x < 3, x \in R$

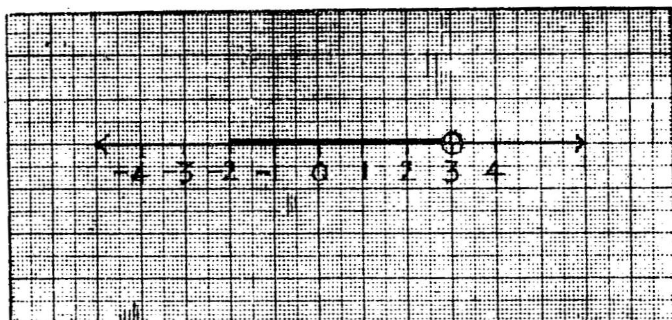


Fig. 6 - 25.

Find out the inequality as well as the solution set of the following :

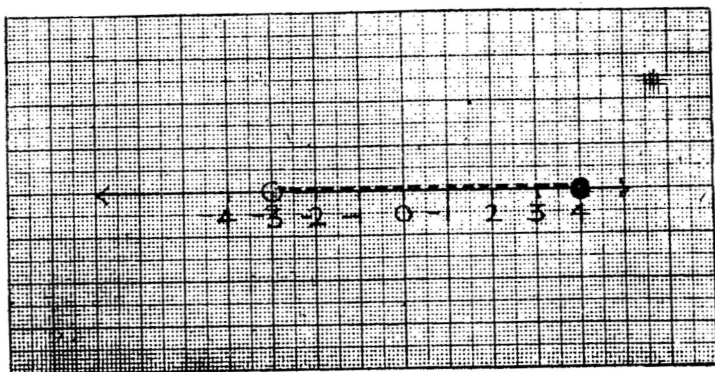


Fig. 6 - 26.

Inequality $-3 < x \leqslant 4, x \in Q$.

Exercise 4—2

1. Draw the graphs of the following inequalities taking the solution sets to be in (i) the set of real numbers (ii) the set of rational numbers (iii) the set of integers :

(a) $-3 \leqslant x \leqslant 4$ (b) $2 \leqslant x < 5$ (c) $-3 < x \leqslant 0$
(d) $-1 < x < 3$.

2. Find out the inequalities representing the following graphs. Find also the sets on which they are defined.

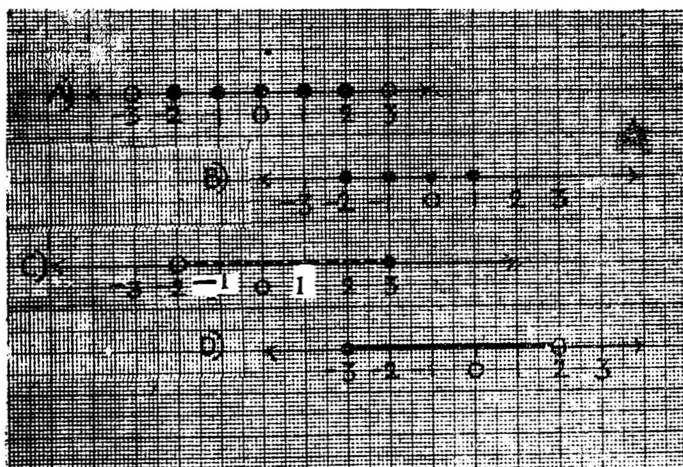


Fig. 6 - 27.

Answers

2. (a) $-3 < x < 3,\ x \in Z$ (b) $-2 \leq x \leq 1,\ x \in Z$

(c) $-2 < x \leq 3,\ x \in Q$ (d) $-3 \leq x < 2,\ x \in R.$

4—3. Inequalities : $ax + by > c,\ ax + by < c$

We know that every line divides the plane into two parts.

Let us take the graph of $2x + 3y = 6$.

The plane (Fig. 6-28) now consists of half plane I, half plane II and the line. Every point on the line will satisfy the equation $2x + 3y = 6$.

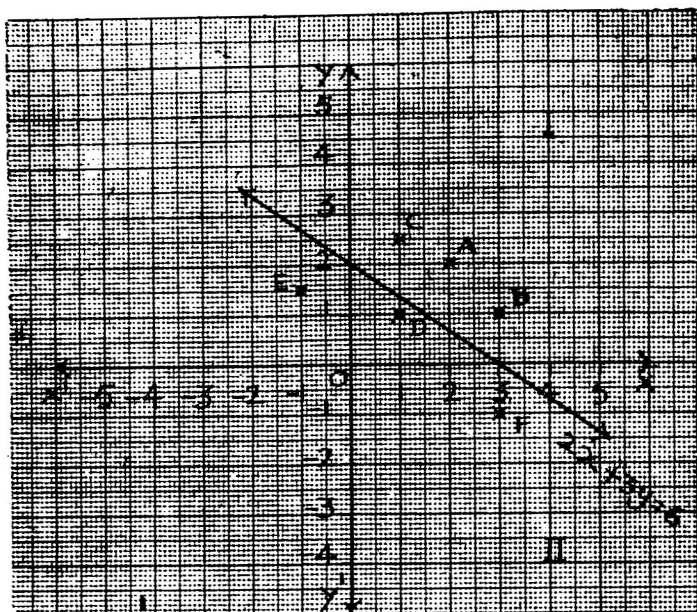


Fig. 6 - 28.

The co-ordinates of the points A, B, C in the I half plane are (2, 2), (3, 1) and (1, $2\frac{1}{2}$) respectively.

The co-ordinates of the points D, E, F in the II half plane are (1, 1), (– 1, $1\frac{1}{2}$), (3, —1) respectively.

Let us fill up the following table :

Plane	Point	x	y	2x + 3y	Inequality
I	A	2	2	4 + 6 = 10	$2x + 3y > 6$
I	B	3	1	6 + 3 = 9	$2x + 3y > 6$
I	C	1	2·5	2 + 7·5 = 9·5	$2x + 3y > 6$
II	D	1	1	2 + 3 = 5	$2x + 3y < 6$
II	E	—1	1·5	—2 + 4·5 = 2·5	$2x + 3y < 6$
II	F	3	—1	6 — 3 = 3	$2x + 3y < 6$

Take some more points and find the inequalities.

From this we see that every point in the I half plane satisfies the inequality $2x + 3y > 6$ and every point in the II half plane satisfies the inequality $2x + 3y < 6$.

Hence we know that,

1. The solution set of $2x + 3y > 6$ lies in the I half plane
2. The solution set of $2x + 3y < 6$ lies in the II half plane
3. The solution set of $2x + 3y = 6$ lies on the line.

Let us shade the solution sets in the graph.

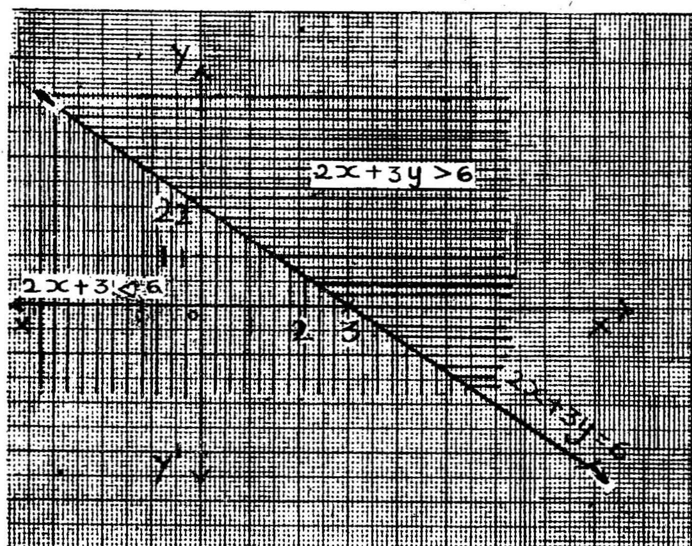


Fig. 6 - 29.

Exercise 4—3

Draw the graphs of the following inequalities :

(1) $3x + 2y > 6$ (2) $2x + 5y > 10$

(3) $4x + 3y > 24$ (4) $3x + 2y < 6$

(5) $2x + 5y < 10$ (6) $4x + 3y < 24$

4—4. Inequalities : $y > kx$; $y < kx$

Take the graph of $y = 2x$ and plot the points in the two half planes. After finding their coordinates find the inequalities satisfying the points in both the half planes.

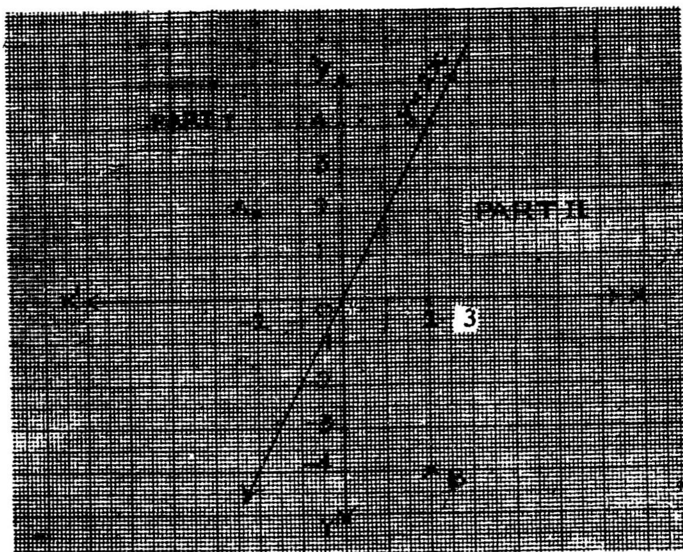


Fig. 6 - 30.

A $(-2, 2)$ is a point in the I half plane.

Hence $x = -2$, $y = 2$. $2x = -4$, $2 > -4$ $\therefore y > 2x$.

B $(2, -4)$ is a point in the II half plane.

Hence $x = 2$, $y = -4$. $2x = 4$, $-4 < 4$ $\therefore y < 2x$.

For finding the inequality, it is enough if we take a point in any one of the half planes.

Taking the graph of $y = -2x$, let us shade the half planes denoting $y > -2x$ and $y < -2x$ respectively.

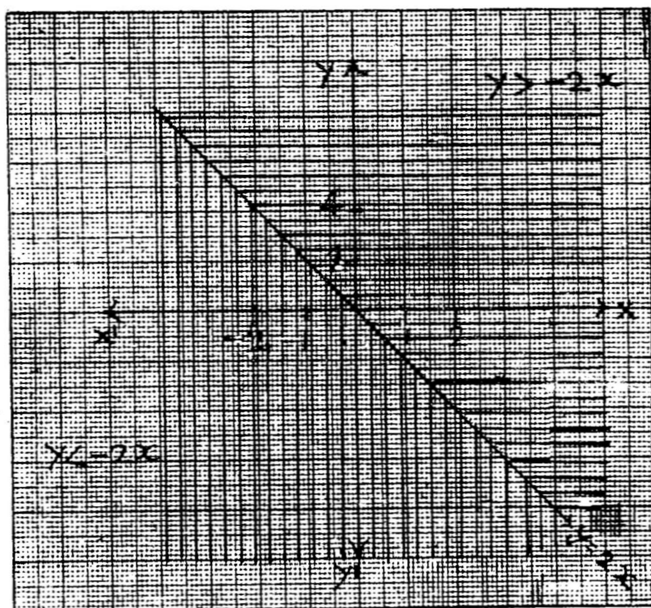


Fig. 6-31.

Exercise 4—4

Draw the graphs of the following :

(1) $y > -3x$ (2) $y < -3x$ (3) $y > 3x$

(4) $y < 3x$.

4—5. Inequalities : $ax + by \geqslant c$, $ax + by \leqslant c$

The inequality $ax + by \geqslant c$ is formed with the help of the equations $ax+by = c$ as well as the equation $ax + by > c$.

Hence the graph of $ax + by \geq c$ includes the line $ax + by = c$ and the half plane representing $ax + by > c$. The line will be thick.

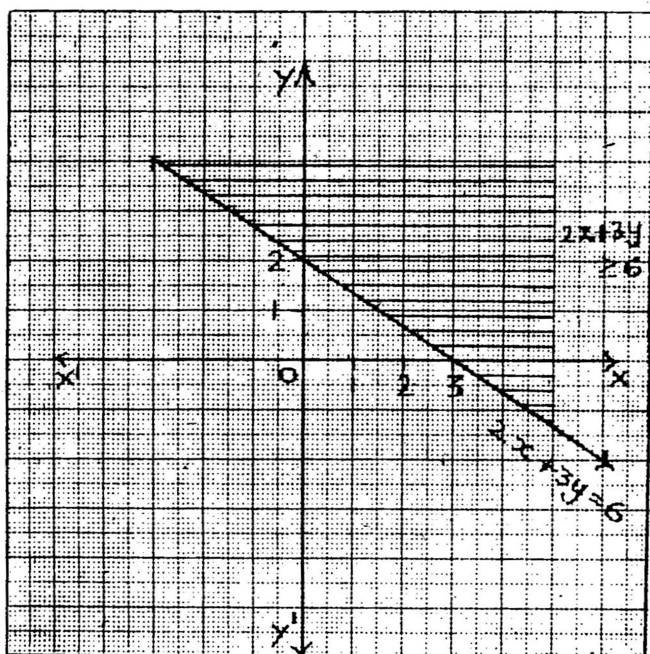


Fig. 6-32.

The shaded part in Fig. 6 - 32 represents the inequality $2x + 3y > 6$.

In $2x + 3y > 6$ we have a broken line. It denotes that the line is not a part of the solution set.

Fig. 6 - 33 is the graphical representation of the inequality $2x + 3y \leq -6$.

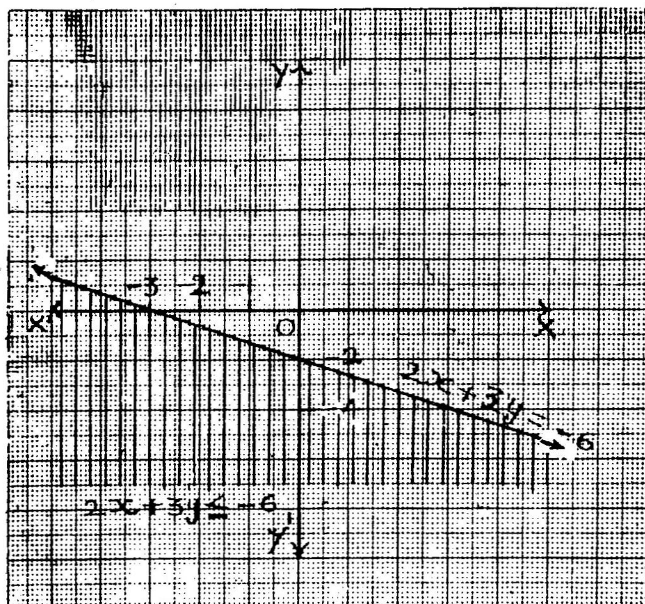


Fig. 6 - 33.

Exercise 4—5

Draw the graphs of the following inequalities :

(1) $3x + 2y \geqslant 6$ (2) $2x + 5y \geqslant 10$ (3) $4x + 3y \geqslant 24$

(4) $3x + 2y \geqslant -6$ (5) $2x + 5y \geqslant -10$ (6) $3x + 2y < 6$

(7) $2x + 5y \leqslant 10$ (8) $4x + 3y \leqslant 24$ (9) $3x + 2y \leqslant -6$

(10) $2x + 5y \leqslant -10$.

5. Statistical graph — Circular graph (pie diagram)

We know that,

$$\frac{\text{Area of a sector}}{\text{Area of the circle}} = \frac{\text{Central angle}}{360}$$

If a circle is divided into many sectors, the ratios of their areas will be in the ratios of the central angles.

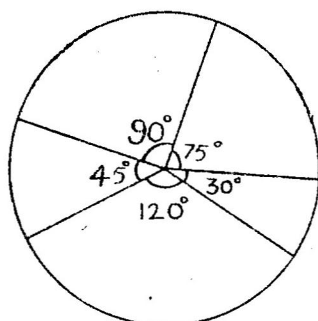


Fig. 6 - 34.

In the figure the circle is divided into five parts.

If the angles subtended by them at the centre are 75°, 90°, 45°, 120° and 30° respectively, then the areas will be in the ratio 75 : 90 : 45 : 120 : 30.

Making use of this we can express the given data by drawing the diagram.

Example :

25% of the boys of a school are in scouting, 15% in N.C.C. and 40% in Red Cross and other movements. Others are not in any movement. Represent the above data by a pie diagram.

Total number of students 100%

100% denotes 360° ; 1 percent denotes 3·6°.

Scouts	25%	25 × 3·6	90°
N.C.C.	15%	15 × 3·6	54°
Other movements	40%	40 × 3·6	144°
Others	20%	20 × 3·6	72°
Total	100%		360°

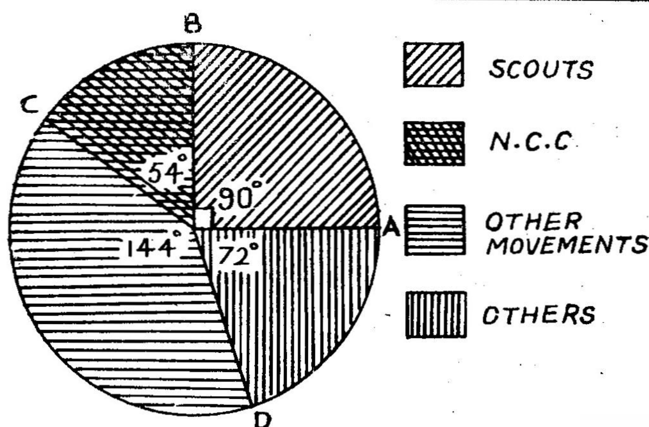


Fig. 6 - 35.

Exercise 5

Represent the following data by pie diagrams :

1. The contents of a food stuff are as follows :
   carbo hydrate 40% ; protein 45%; fat 10%; vitamin 5%.
2. Areas of oceans (in square kilo metre):
   Pacific 170; Atlantic 90; Indian ocean 65; Antarctic 20; Arctic 15.
3. The expenditure of a family are as follows :
   Food Rs. 160; Dress Rs 80; Rent Rs. 60; Fuel Rs. 20; Other items Rs. 80.
4. A rupee on 'Khadi' is distributed as follows :
   Farmer 19 p; Spinner 35 p; Weaver 28 p; Washerman and dyer 8 p; Administrative agency 10 p.

MATHEMATICS CLUB—ACTIVITY 6

Puzzle

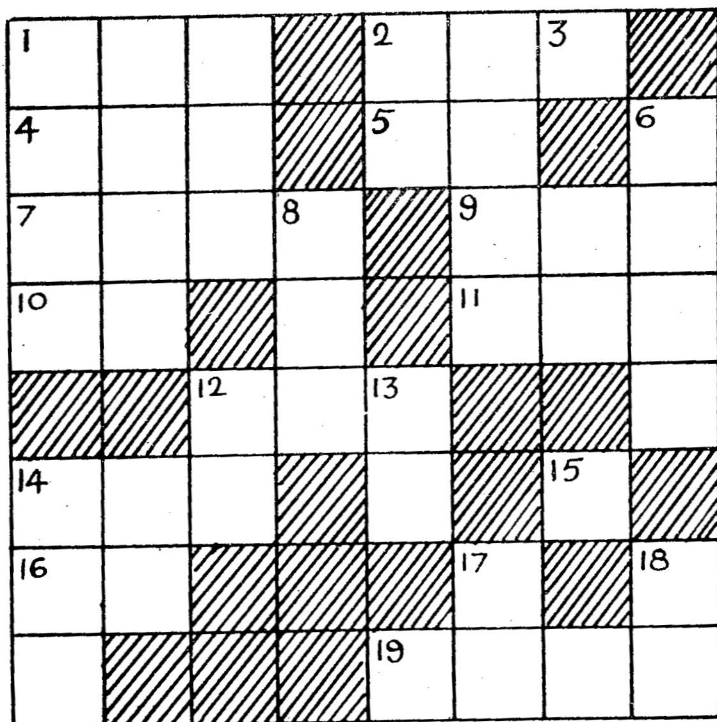


Fig. 6-36.

Across

1. 15^2
2. (5 + 9) (8 + 7)
4. What is x^3, if $x = 7$?
5. Number of two digit odd numbers
7. 15 in binary numeration
9. 1st, 2nd, 3rd powers of 2
10. Simple interest for Rs. 500 at 10% for 18 months
11. $3^3 \times 2^2$
12. 3^5
14. 24 dozens
16. 1101_2 in decimal numeration
19. $12^3 + 1^3 = 10^3 + 9^3$

Down

6. Year of birth of Ramanujam
8. 44 in quenary numeration
12. A perfect number
13. Area of a right triangle with sides 9 and 8 forming a right angle
14. Compound interest for Rs. 1000 at 10% for 2 years
15. Contribution of Indians to numeration
17. The greatest 2 digit prime
18. $\sqrt{9801}$

7. APPLICATIONS

1. Direct Hire Purchase

Sometimes we may not be able to pay the entire price of an article such as fan, radio, T. V. set, wet grinder, vessels, houses, plots (for houses) etc., at the time of purchase. In these situations we may pay a part of the cost at the time of purchase and agree to pay the balance or sometimes the whole cost in easy monthly instalments.

You would have seen such advertisements in dailies. 'Plots for houses—Nehru Nagar—Chromepet—Rs. 3500 per ground—Rs. 500 at the beginning—the balance in easy monthly instalments.'

While paying in instalments, interest will be charged for the amount due. Since the amount due decreases month by month in case of monthly instalments, the interest will also decrease in successive months. Let us work out such problems taking simple interest into consideration.

Example 1 :

The cost of a cycle is Rs. 500. If we get in hire purchase scheme, we have to pay a simple interest of 15% per annum. If we pay in 12 instalments, find the monthly instalment.

The cost of a cycle is Rs. 500

Interest 15 %

Amount due with interest $= \text{Rs. } \dfrac{500 \times 115}{100} = \text{Rs. } 575$

Let the monthly instalment be Rs. x

Amount paid in 12 months = Rs. 12x

Interest earned by the amount paid

$$= \text{Rs. } x \times \frac{12 \times 13}{2 \times 12} \times \frac{15}{100} = \text{Rs. } \frac{39x}{40}$$

Total amount paid $= 12x + \frac{39\,x}{40} =$ **Rs.** $12\frac{39}{40}\,x$

$\therefore \quad 12\frac{39}{40}\,x = 575$

$$x = \frac{575 \times 40}{519} = \text{Rs. } 44\text{-}32$$

The monthly equivalent payment will be Rs, 44–32.

Example 2:

The cost of a T. V. Set is Rs. 4200. If we want to buy it in monthly instalments we have to pay Rs. 1200 at the beginning and the balance in 30 instalments of Rs. 120 each. If the simple interest calculated is 18% find the amount paid? How much excess has been paid?

Amount due $= \text{Rs. } 4200 + \text{Rs. } 4200 \times \frac{30}{12} \times \frac{18}{100}$

$= \text{Rs. } 6090$

Amount paid at the beginning $= \text{Rs. } 1200$

Interest earned for 30 months $= \text{Rs. } 1200 \times \frac{30}{12} \times \frac{18}{100} = \text{Rs. } 540$

Amount paid in 30 instalments of Rs. 120 each $= \text{Rs. } 3600$

Interest earned for the instalment payments $= \text{Rs. } 120 \times \frac{30 \times 31}{2 \times 12} \times \frac{18}{100}$

$= \text{Rs. } 837$

Total amount paid $= \text{Rs. } 1200 + \text{Rs. } 540 + \text{Rs. } 3600 + \text{Rs. } 837$

$= \text{Rs. } 6177$

Amount due $= \text{Rs. } 6090$

Excess paid $= \text{Rs. } 87$

Exercise 1

1. The price of a fan is Rs. 420. A man agreed to pay the cost in 40 instalments with 18% simple interest. Calculate the monthly instalment.

2. The cost price of a wet grinder is Rs. 1500. A man purchased it agreeing to pay its cost in 20 equal instalments with 15% simple interest. Calculate the monthly instalment.

3. An oil crushing machine costs Rs. 4000. A man paid Rs. 1000 and agreed to pay the balance in 15 equal instalments at 15% simple interest. What is the total amount paid ? How much is paid as interest ?

4. The cost price of a radio set is Rs.600. A man purchased it agreeing to pay in 12 equal instalments with 12% simple interest. Find the monthly instalment.

Answers

1. Rs. 44-62 2. Rs. 89-88 3. Rs. 4431-25, Rs. 431-25 4. Rs. 42-59.

2—1. Compound Interest — Revision

We have learnt in the earlier standards, what compound interest is and how to compute the compound interest and the amount.

Notation used in compound interest :

P — principal

n — time

i — rate of interest

Formula to find the amount in compound interest is

$$A = P(1 + i)^n$$

Example :

Compute the compound interest on Rs. 500 for 2 years at 5%.

This can be done in two ways.

Formula method :

$$A = P(1 + i)^n$$
$$= Rs.\ 500\ (1 + 0{\cdot}05)^2$$
$$= Rs.\ 500 \times 1{\cdot}05 \times 1{\cdot}05$$
$$= Rs.\ 551.25$$

Amount = Rs. 551.25

Interest = Rs. 551.25 — Rs. 500

= Rs. 51.25

Aliter :

Principal for the 1st year = Rs. 500

Interest = Rs. 25

Principal for the 2nd year = Rs. 525.00

Interest = Rs. 26·25

Amount = Rs. 551.25

Principal = Rs. 500.00

Interest = Rs. 51.25

Note : For rate of interest if we use R% instead of i, then the formula for finding the amount is $A = P\left(1 + \frac{R}{100}\right)^n$

Exercise 2—1

Find the compound interest and the amount :

	Principal	Rate of interest	Time
1.	Rs. 2000	5%	2 years
2.	Rs. 2500	8%	2 years
3.	Rs. 3600	9%	2 years
4.	Rs. 1400	12%	2 years
5.	Rs. 5000	15%	2 years

Answers

1. Rs. 2205, Rs. 205
2. Rs. 2916, Rs. 416
3. Rs. 4277-16, Rs. 677-16
4. Rs. 1756-16, Rs. 356-16
5. Rs. 6612-50, Rs. 1612-50

2—2. Compound Interest (a) — 3 Terms

Let us now see how to calculate compound interest for three years.

Example 1 :

Calculate the compound interest on Rs. 2000 for 3 years at 9% interest.

Principal for the 1st year = Rs. 2000
Interest thereon = Rs. 180.00

Principal for the 2nd year = Rs. 2180·00
Interest thereon = Rs. 196·20

Principal for the 3rd year = Rs. 2376·20
Interest thereon = Rs. 213·8580

Amount = Rs. 2590·0580
= Rs. 2590·06
Interest = Rs. 590·06

Aliter :

Using the formula

$$A = P(1 + i)^n$$
$$= 2000\,(1 + 0\cdot09)^3$$
$$= 2000 \times 1\cdot09 \times 1\cdot09 \times 1\cdot09$$
$$= \text{Rs. } 2590.06$$

Amount = Rs. 2590.06
Interest = Rs. 590.06

Example 2 :

Compute the C. I. on Rs. 3000 for $2\frac{1}{2}$ years at 8% interest.

Principal for the 1st year = Rs. 3000
Interest = Rs. 240

Principal for the 2nd year = Rs. 3240.00
Interest = Rs. 259.20

Principal for the 3rd year = Rs. 3499-20
Interest for the $\frac{1}{2}$ year = Rs. 139-9680

Amount = Rs. 3639-1680

= Rs. 3639-17

Interest = Rs. 639-17

(Note : Interest for $\frac{1}{2}$ year is 4%)

Exercise 2—2

Find the amount and the C. I.

	Principal	Rate of Interest	Time
1.	Rs. 4000	9%	3 years
2.	Rs. 6000	10%	3 years
3.	Rs. 8000	11%	3 years
4.	Rs. 12000	12%	3 years
5.	Rs. 5000	8%	$2\frac{1}{2}$ years
6.	Rs. 10000	10%	$2\frac{1}{2}$ years
7.	Rs. 15000	12%	$2\frac{1}{2}$ years

Find the C.I. if the interest is compounded every 6 months.

8.	Rs. 9000	12%	$1\frac{1}{2}$ years
9.	Rs. 11000	10%	$1\frac{1}{2}$ years
10.	Rs. 20000	15%	$1\frac{1}{2}$ years

Answers

1. Rs. 5180-12, Rs. 1180-12
2. Rs. 7986, Rs. 1986
3. Rs. 10941-05, Rs. 2941-05
4. Rs. 16859-14, Rs. 4859-14
5. Rs. 6065-28, Rs. 1065-28
6. Rs. 12425, Rs. 2425
7. Rs. 19944-96, Rs. 4944-96
8. Rs. 10719-14, Rs. 1719-14
9. Rs. 12733-88, Rs. 1733-88
10. Rs. 24845-94, Rs. 4845-94.

2—3. Compound Interest (b)

Example 1 :

A man invests Rs. 2000 at the beginning of every year at 6% interest. Find the amount to his credit at the end of the third year.

Principal at the beginning of the 1st year = Rs. 2000
Interest = Rs. 120
Amount invested at the beginning of the 2nd year = Rs. 2000
Principal for the 2nd year = Rs. 4120
Interest = Rs. 247.20
Amount invested at the beginning of the 3rd year = Rs. 2000
Principal for the 3rd year = Rs. 6367-20
Interest = Rs. 3 82-0320
Amount = Rs. 6749-2320
= Rs. 6749-23

Example 2 :

Calculate the amount and compound interest on Rs. 6000 for 3 years at $7\frac{1}{2}$ % interest.

$$7\tfrac{1}{2} = 5 + 2\tfrac{1}{2}$$

$$5\% = \tfrac{1}{20};\ 2\tfrac{1}{2}\% = \tfrac{1}{2} \times 5\%$$

Principal for the 1st year = Rs. 6000
Interest at 5% = Rs. 300
Interest at $2\frac{1}{2}$% = Rs. 150

Principal for the 2nd year = Rs. 6450
Interest at 5% = Rs. 322.50
Interest at $2\frac{1}{2}$% = Rs. 161.25

Principal for the 3rd year = Rs. 6933.75

Interest at 5% = Rs. 346.6875

Interest at $2\frac{1}{2}$% = Rs. 173.34375

Amount = Rs. 7453.78125

= Rs. 7453.78

Interest = Rs. 1453.78

By using formula :

$$A = P \left(1 + \frac{R}{100}\right)^n$$

$$= \text{Rs. } 6000 \left(1 + \frac{15}{2 \times 100}\right)^3$$

$$= \text{Rs. } 6000 \times \frac{43}{40} \times \frac{43}{40} \times \frac{43}{40}$$

= Rs. 7453.78

Interest = Rs. 1453-78

Example 3 :

Calculate the amount on Rs. 5000 interest compounded once in 6 months for $1\frac{1}{2}$ years.

Principal for the 1st 6 months = Rs. 5000

Interest = Rs. 200

Principal for the 2nd 6 months = Rs. 5200

Interest = Rs. 208

Principal for the 3rd 6 months = Rs. 5408

Interest = Rs. 216.32

Amount = Rs. 5624.32

Amount = Rs. 5624.32

Interest = Rs. 624.32

[Note : Rate of interest for 6 months = $\frac{1}{2}$ × 8% = 4%. Hence multiply the principal by 0·04]

By using formula :

$$P = \text{Rs. } 5000; \quad i = \frac{1}{2} \times \cdot 08 = \cdot 04; \; n = \frac{18}{6} = 3$$

$$A = 5000\,(1 + 0{\cdot}04)^3$$
$$= 5000 \times 1{\cdot}04 \times 1{\cdot}04 \times 1{\cdot}04$$
$$= \text{Rs. } 5624.32$$
$$\text{Interest} = \text{Rs. } 624.32$$

Example 4 :

The cost of a machine is Rs. 20000. Its depreciation value is 5% per annum. Find its value after 3 years.

The initial value of the machine	= Rs. 20000
Depreciation for the 1st year	= Rs. 1000
The value of the machine at the beginning of the 2nd year	= Rs. 19000
Depreciation for the 2nd year	= Rs. 950
The value of the machine at the beginning of the 3rd year	= Rs. 18050
Depreciation for the 3rd year	= Rs. 902.50
The value of the machine at the end of the 3rd year	= Rs. 17147.50

[Note : Depreciation is calculated on the value of the machine at the beginning of the year.]

This can be done by using the formula also.

$$A = P\,(1 + i)^n$$
$$= 20000\,[\,1 + (-0{\cdot}05)\,]^3$$
$$= 20000 \times {\cdot}95 \times {\cdot}95 \times {\cdot}95$$
$$= \text{Rs. } 17147{\cdot}50$$

Exercise 2—3

1. A man invests Rs. 3000 at the beginning of every year at 6% C.I. Find the amount to his credit at the end of the 3rd year ?

2. A business man deposits Rs. 4000 at the beginning of every year at 8% C.I. in a bank. Find the amount to his credit at the end of $2\frac{1}{2}$ years.

3. A business man deposits Rs. 15000 in a bank at 9% C.I. and withdraws Rs. 5000 at the end of every year. How much will he get at the end of the 3rd year ?

4. The population of a city increases by 5% every 10 years. If the population of the city is 640000 at the beginning of 1940, find its population at the beginning of 1970.

5. The depreciation value of a machine is 10% per year. Find the value of the machine after 3 years if the present value is Rs. 12000.

6. The depreciation value of a machine is 6% per year. A machine is bought for Rs. 25000. Find its value after 3 years.

Answers

1. Rs. 10123-85	2. Rs. 13494-62	3. Rs. 3034-94
4. Rs. 740880	5. Rs. 8748	6. Rs. 20764-60

MATHEMATICS CLUB — ACTIVITY 7

1. Anbu, Alagan and Arivu study in different schools. Tomorrow they will have a test.

Anbu does not study in Mani High School. Alagan does not read in Sarva Jana High School. Mani High School pupils will not have Mathematics test. Sarva Jana High School pupils will have English test. Alagan will not have a Science test. In which schools do they study? What test will they be writing? Do you want the name of the third School? Let it be the name of your own school.

2. I have some hens and sheep. The total number of legs is 20. How many sheep do I have if the number of heads is 8 ?

8. MATHEMATICAL LOGIC

1—1. Mathematical Statements

(a) Look at the following sentences ;

(1) India got freedom in the year 1947

(2) India got freedom in the year 1857

(3) When did India get freedom ?

We can see that the first sentence is true, the second one is false and we cannot say anything definitely about the third sentence. The first and the second sentences are known as statements. A statement is either true or false.

$8+5=13$; $3\times5=35$; $9-4=6$; $15\div3=5$ are some examples of statements. Of these $8+5=13$; $15\div3=5$ are true statements whereas $3\times5=35$ and $9-4=6$ are false statements.

Can you say whether the statement $x+5=13$ is true or false ? This question can be answered only if we know what value we give for x. We know that $x+5=13$ is true for $x=8$ and is false for all other values of x.

The sentences of the type $x+5=13$ are known as open sentences.

(b) Quantifiers : All, every, some.

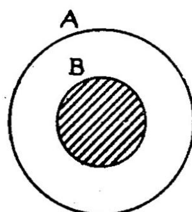
$x+5<13$ is an open sentence. It is true for all values of x. If it takes values in the set $\{-3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7\}$ then $x+5<13$ is true for every x if $x \in \{x/x<8\}$. The words "for every x " and 'for all x' give the same meaning.

The statements 'All the Tamilians are Indians' and 'Every Tamilian is an Indian' mean the same thing.

We can say that if one is not an Indian, then he cannot be a Tamilian. Can we say that the converse is true ? ' Some Indians are Tamilians' is a true statement. ' Some Indians

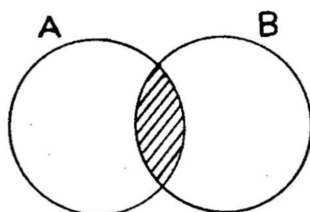
are Tamilians' means that there are some Indians other than Tamilians.

This can be explained through a Venn Diagram.



$B \subset A$

Fig. 8-1.



A and B are not disjoint sets.

Fig. 8-2.

(1) All B's are A's. Every B is A. There is no B which is not A.

(2) Some A's are B's. Some B's are A's. There are some A's other than B's. There are some B's other than A's.

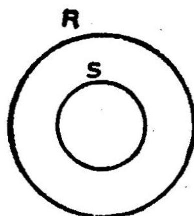
Examples :

(1) 'All squares are rectangles.' Some equivalent sentences are given below :

(a) If a figure is a square, then it is a rectangle.

(b) Every square is a rectangle.

(c) There is no square which is not a rectangle.

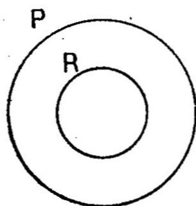


Set R : Rectangles

Set S : Squares

Fig. 8-3.

(2) 'Some parallelograms are rectangles'. An equivalent sentence for this is, 'there exists at least one parallelogram which is not a rectangle'.

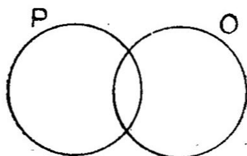


Set P : Parallelograms

Set R : Rectangles

Fig. 8-4.

(3) 'Some prime numbers are odd numbers.' It implies that there exists at least one prime number which is not an odd number. Can you give an example for it ?



P : Set of prime numbers

O : Set of odd numbers

Fig. 8-5.

Verify that the above figure holds good also for the sentence "Some odd numbers are prime numbers' . Give an example for even prime number.

Exercise 1—1

State equivalent sentences for the following sentences. Draw Venn Diagrams.

(1) Measure of every angle of a rectangle is 90°.

(2) Some rhombuses are squares.

(3) Some multiples of 4 are multiples of 3.

(4) There exists at least one multiple of 5 which is not an odd number.

(5) There is 10th std. in all High schools.

(6) There is no boy who does not wear a blue shirt.

1—2. Compound Sentences

The combination of more than one sentence is known as a compound sentence. We come across different kinds of compound sentences in mathematical logic. Let us see the kinds of compound sentences.

Look at the following sentences :

p : Today is a holiday.

q : I will be at home.

Some compound sentences got by combining the above sentences are given below :

(1) Today is a holiday and I will be at home.

(2) Today is a holiday or I will be at home.

(3) If today is a holiday then I will be at home.

(a) The compound sentence got by using the word 'and' is true only if both the given sentences are true. If one of the given sentences is false, then the compound sentence will not be true.

(b) The compound sentence got by using the word 'or' is true if either of the given sentences is true. If both of them are false, then the compound sentence got is also false.

(c) The compound sentence got by using the words 'If.........then' is known as 'conditional sentence'. It is denoted by $p \rightarrow q$. It can be written as "if p, then q". In "If p, then q" p is known as antecedent and q is known as consequent. This is very much used in mathematical logic.

(d) Note : If 'p: today is a holiday' is a statement, then its negation is 'Today is not a holiday'. It is denoted either by $\sim p$ or by 'not p'.

If p is true, $\sim p$ is not true.

If p is not true, $\sim p$ is true.

2—1. Mathematical Systems

We know that "the sum of the three angles of a triangle is two right angles". You got this fact by measuring the angles of several triangles.

Here is an example which warns us against making generalisations from some specific examples.

Is n^2+n+41 a prime number for all $n \in N$?

n	1	2	3	4	5	6	7	8	9	10
n^2+n+41	43	47	53	61	71	83	97	113	131	151

From the above we are likely to conclude that n^2+n+41 always gives a prime number. This is not correct. If we take $n=40$ then $n^2+n+41=1681=41 \times 41$. We find therefore n^2+n+41 does not represent a prime number for all $n \in N$. Similarly we cannot accept any property found in some specific examples as a generalised fact. A sentence is to be proved to be true or false only through mathematical logic.

We build up a massive structure of mathematical facts, known as theorems. using (a) some undefined terms as point, line, plane (b) some postulates and axioms and (c) some terms or definitions based on those undefined terms and axioms. This constitutes a mathematical system. For different terms and axioms we come across different mathematical systems.

Greeks applied mathematical logic to prove geometrical facts 2500 years ago. Euclid collected more then 400 theorems and published his famous books, in 300 B.C. The proofs given in Euclid fills us with wonder even today. Euclid took the following five postulates :

(1) A line can be drawn joining two points

(2) A line has no starting point and no end point

(3) A circle can be drawn by taking a point as its centre for given radius

(4) Right angles are equal to one another

(5) If the sum of the interior angles formed by a transversal cutting two lines is less than two right angles, then these lines will intersect at a point.

The Euclidian geometry based on the above five postulates is a wonder to the world. Some new geometries have been developed by modifying these five postulates. The geometries invented by Lobachevski, Riemann and Bolyai have created new history. You will learn about these geometries in higher mathematics.

2—2. Direct Proof

The words 'if then' are very much used in mathematica logic. In $p \rightarrow q$, p is antecedent or hypothesis and q is consequent or conclusion.

Let us consider $p \rightarrow q$ as a postulate or an accepted fact If p is true, then q is also true.

Direct method of proving facts;

Postulate ; $p \rightarrow q$ (if p, then q)

Given : p is true.

Conclusion : q is true.

Examples :

1. **Axiom :** Those who get the first mark will get scholarships.

Given : Kuppan got the first mark.

Conclusion : Kuppan will get the scholarship.

2. **Axiom :** The sum of the angles of a triangle is 180°

Given : ABC is a triangle.

Conclusion : $m\angle A + m\angle B + m\angle C = 180°$.

If $p \rightarrow q$ and $q \rightarrow r$ then $p \rightarrow r$.

3. **Axioms :** (i) If square, then it is rectangle, $p \rightarrow q$.

(ii) If rectangle, then it is quadrilateral, $q \rightarrow r$.

Given : ABCD is a square.

Conclusion : $p \rightarrow q$ and $q \rightarrow r$

$\therefore \; p \rightarrow r$.

If square then it is quadrilateral.

Defined terms :

Centre, radius, line, circumference, chord, Right angle, triangle, perpendicular, hypotenuse, equal, congruent.

Axiom : If the hypotenuse and one side of a right angled triangle are equal to the hypotenuse and the corresponding side of another right angled triangle, then the two triangles are congruent.

Using these we can prove the following theorem :

Theorem : The line drawn perpenticular to a chord of a circle from its centre bisects the chord.

Data : Let O be the centre and OA be the radius of the circle. AB is a chord.

$OC \perp AB$.

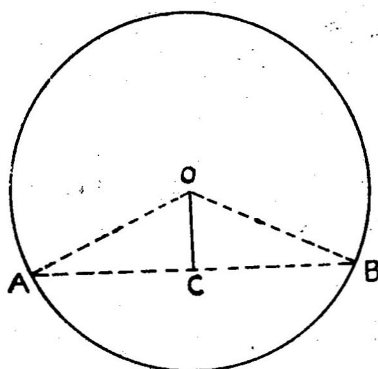


Fig. 8-6.

To prove that : $\overline{AC} = CB$

Proof : Join OA and OB (Axiom)

In $\triangle$ OAC and $\triangle$ OBC

m $\angle OCA$ = m $\angle OCB$ = 90° (data)

Radius $\overline{OA}$ = Radius $\overline{OB}$ (defined term)

$\overline{OC} = OC$

$\therefore$ $\triangle OAC \equiv \triangle OBC$ (Axiom)

$\therefore$ $\overline{AC} = \overline{BC}$ (defined term)

2—3. Indirect method of proving facts

There are three kinds of indirect proofs :

(a) Method of elimination

(b) Giving counter examples

(c) Proving the contra-positive

Method of Elimination

Consider that an article was stolen from a house and only four persons used to visit that house. A has gone out when the theft took place; B did not come out of his house on that day; C is a follower of Gandhi. Therefore we can conclude that A, B, C are not thieves and the fourth one is the person who committed the theft.

We can give one example for this method in geometry.

Axioms :

(1) If two angles of a triangle are equal, then their opposite sides are also equal.

(2) If two sides of a triangle are not equal, then the angle opposite the bigger side is greater than the angle opposite the smaller side.

To be proved :

If two angles of a triangle are not equal, then the side opposite the greater angle is bigger than the side opposite the smaller angle.

Proof :

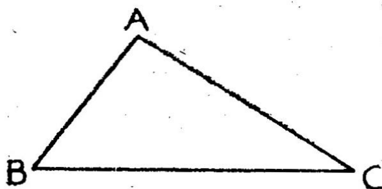


Fig. 8-7.

Given :

In $\triangle$ ABC, $m\angle B > m\angle c$,

To prove that: $\overline{AC} > \overline{AB}$

Proof :

If AC = AB then m $\angle$ B = m $\angle$ C (Axiom 1)

It violates the hypothesis.

Therefore $\overline{AC} \angle$ AB.

If $\overline{AC} < \overline{AB}$ then AB > $\overline{AC}$

i. e., m$\angle$C > m$\angle$B (Axiom 2)

This also violates the hypothesis. AB > $\overline{AC}$

$\therefore$ $\overline{AC} > \overline{AB}$.

Note the following :

$\overline{AB} = \overline{AC}$, AB > $\overline{AC}$ and $\overline{AC} > \overline{AB}$ are the only three possibilities. Out of these we eliminate two possibilities as they contradict the hypothesis and conclude that the third possibility holds good.

(b) Disproving through counter example

$n^2 + n + 41, n \in N$ is a prime number. This statement is disproved by giving one counter example.

The statement "Any quadrilateral with all sides congruent is a square" can be disproved by giving a counter example, namely a Rhombus.

The statement "All prime numbers are odd numbers" can be disproved by giving a counter example, namely 2.

(c) Proving the contra-positive

In $p \rightarrow q$, if q is not true, we can prove that p is also not true.

That is, $p \sim q$ and $\sim q \rightarrow \sim p$ are two statements having the same meaning.

Example :

$a \in Z$, if a^2 is an even number, then a is also an even number.

q : a is not an even number.

i.e. a is an odd number.

If a is an odd number, then a^2 is also an odd number. But a^2 is given to be an even number. Therefore $\sim p$ is true. So we conclude a cannot be odd. $\sim p \rightarrow \sim p$ is equivalent to $p \rightarrow q$.

$\therefore$ a is an even number.

Exercise 2—3

1. Some axioms and some data are given below. Find the results which can be obtained from them.

(a) The sum of two sides of a triangle is greater than the third side.

ABC is a triangle.

(b) If a man has completed 21 years, he has got franchise. One who has completed 25 years can be a candidate.

The age of Anbarasan is 24.

(c) The numbers divisible by 12 are divisible by 4.

The last two digits of a number which is divisible by 4 is divisible by 4.

(d) The opposite angles of a cyclic quadrilateral are supplementary

ABCD is a cyclic quadrilateral.

(e) If it is a full moon day we can play "Kilithattu".

Today is a full moon day.

(f) Only males have colour blindness.

She is my mother.

(g) If there is a heavy crowd the film is a good one.

I didn't get the ticket.

(h) $w \rightarrow v$; $u \rightarrow w$; $x \rightarrow u$.

v is not true.

(i) If A is green, B is red.
If A is blue; B is black.
If B is red, Y is white.
A is green.

2. Tell the theorems using the following axioms :

(a) A and B are students
C and D are students
Sister of A is C
Sister of D is B
Brother of A is B
(Find 8 results)

(b) Brother of B is A
Father of A is C

(c) Brother of B is A
Brother of A is C
Brother of C is B
Brother of D is B

MATHEMATICS CLUB—ACTIVITY 8

Solve the following puzzles which have letters instead of digits.

1.
```
   C R O S S
   R O A D S
 -----------
 D A N G E R      (Take S = 3)
```

2.
```
   A A A
   B B B
   C C C
 -------
 B A A C
 -------
```

MATHEMATICS CLUB — ACTIVITY 9

Magic Squares

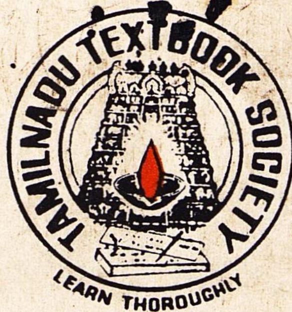
In the following squares the sum of each of the rows, columns and the two corner diegonal column are the same. Next to them can be found the squares explaining the development.

16	2	3	13
5	11	10	8
9	7	6	12
4	14	15	1

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

64	2	3	61	60	6	7	57
9	55	54	12	13	51	50	16
17	47	46	20	21	43	42	24
40	26	27	37	36	30	31	33
32	34	35	29	28	38	39	25
41	23	22	44	45	19	18	48
49	15	14	52	53	11	10	56
8	58	59	5	4	62	63	1

1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48
49	50	51	52	53	54	55	56
57	58	59	60	61	62	63	64



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